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An extension of TOPSIS method using interval bipolar linguistic neutrosophic set and its application

Vu Thi Nhu Quynh[a]

[a]Vietnam Maritime University, 484 Lach Tray, Hai Phong, Vietnam.

Abstract TOPSIS, known as the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), is one of the well-known methods for solving multi-criteria decision-making (MCDM) problems. Many TOPSIS methods have been developed to solve real-life problems using neutrosophic sets. However, no study has developed the TOPSIS method under interval bipolar linguistic neutrosophic environments. Therefore, this study aims to present a new concept of interval bipolar linguistic neutrosophic set (IBL_NS). IBL_NS is more flexible and adaptable to real-world applications than other sets. Some set-theoretic operations, such as union, intersection, and complement, and the operational rules of IBL_NS are defined. Then, a new TOPSIS procedure in IBL_NS is developed. In the proposed IBL_NS-TOPSIS method, ratings of alternatives and importance weights of criteria are expressed in IBL_NS. An application is presented demonstrating the advantages of the proposed IBL_NS-TOPSIS approach.

Keywords: TOPSIS, linguistic neutrosophic set, bipolar neutrosophic set, interval bipolar linguistic neutrosophic set

1. Introduction

Smarandache (1999, 2015) first proposed a neutrosophic set (NS) which is a generalization of fuzzy sets and intuitionistic fuzzy sets. Wang et al (2005) proposed a single-valued neutrosophic set to apply NS in real-life applications. Since then, many studies have applied the single-valued neutrosophic set to various problems in decision-making (Sodenkamp et al 2018; Sert 2018; Singh and Huang 2019; Nasef et al 2020; Abdel-Basset et al 2020; Karamustafa and Cebi 2021; Stanujkić et al 2021; Mishra et al 2021; Dhar and Kundu 2021; Alpaslan 2022; Karadayi-Usta 2022; Tapia et al 2022). However, in many real-life situations, using real numbers in truth, falsehood, and indeterminacy values is inappropriate, which can be expressed as an interval. Wang et al (2005) and Zhang et al (2014) proposed interval-valued neutrosophic sets and their set-theoretic operators. Recently, many studies have applied interval-valued neutrosophic sets to solve real-life problems in literature (Thong et al 2019; Jia et al 2021; Yazdani et al 2021; Torkayesh et al 2022; Pourmohseni et al 2022). Thong et al (2019) presented the new TOPSIS dynamic using dynamic interval-valued neutrosophic sets for evaluating lecturers' performance. Thong et al (2020) proposed an extension of dynamic interval-valued neutrosophic sets to evaluate tertiary students' performance. Jia et al (2021) developed the integrated approach (including interval-valued neutrosophic sets, belief rule base, and Dempster-Shafer evidence reasoning) to form a powerful fault detection algorithm. Yazdani et al (2021) proposed an interval-valued neutrosophic decision-making structure for selecting sustainable suppliers for a dairy company in Iran. Torkayesh et al (2022) introduced a multi-distance interval-valued neutrosophic approach for social failure detection.

However, many studies have shown that linguistic variables are valuable in solving decision-making problems. Several studies have integrated linguistic variables and the concept of interval-valued neutrosophic set in their decision-making models (Ji et al 2018; Garg and Nancy 2019; Lio and You 2019; Zhu et al 2020; Li et al 2022). Li et al (2022) proposed a reliability allocation method based on the linguistic neutrosophic numbers weight Muirhead mean operator. Zhu et al (2020) developed a hybrid risk ranking model of failure mode and effect analysis by combining linguistic neutrosophic numbers, regret theory, and PROMETHEE (Preference ranking organization method for enrichment evaluation) approach. Garg and Nancy (2019) presented the possible linguistic single-valued neutrosophic set for dealing with imprecise and uncertain information during decision-making. Liu and You (2019) presented an approach to determine the distance measure between two linguistic NSs. Ji et al (2018) developed the multi-attribute border approximation area comparison (MABAC) - ELECTRE (the elimination and choice translating reality) method under single-valued neutrosophic linguistic environments.

Deli et al (2015) further presented the concept of bipolar neutrosophic sets based on positive and negative effects in a vague environment. Abdel-Basseta et al (2019) proposed the cosine and weighted cosine similarity measures to rank bipolar

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and IVB_NS. Abdel-Basset et al (2020) developed a hybrid neutrosophic MCDM for chief executive officer selection. Garai and Garg (2022) developed an MCDM approach based on the ranking interpreter for selecting COVID-19 vaccines. Jamil et al (2022) applied Einstein operations to bipolar neutrosophic aggregation operators. It seems that no study has developed the MCDM method under interval bipolar linguistic neutrosophic environments. Therefore, this study presents new concepts of interval bipolar linguistic neutrosophic set (IBL_NS). IBL_NS is more flexible and adaptable to real-world applications than other sets. Some set-theoretic operations, including union, intersection, and complement, and the operational rules of IBL_NS are defined. Then, a new TOPSIS procedure in IBL_NS is developed. In the proposed IBL_NS-TOPSIS method, the ratings of alternatives and importance weights of criteria are expressed in IBL_NS (see Supplementary Material). An application is presented illustrating the advantages of the proposed IBL_NS-TOPSIS approach.

2. Proposed new interval bipolar linguistic neutrosophic set

Definition 1: An IBL_NS $\breve{Z}$ in *X* is defined as the following:

$$\breve{Z}=\left\{\left\langle \begin{array}{l}\breve{\psi},\breve{s}_{\partial_{\breve{Z}}(\breve{\psi})},[T^{L+}_{\breve{Z}}(\breve{\psi}),T^{R+}_{\breve{Z}}(\breve{\psi})],[I^{L+}_{\breve{Z}}(\breve{\psi}),I^{R+}_{\breve{Z}}(\breve{\psi})],[F^{L+}_{\breve{Z}}(\breve{\psi}),F^{R+}_{\breve{Z}}(\breve{\psi})],\\ [T^{L-}_{\breve{Z}}(\breve{\psi}),T^{R-}_{\breve{Z}}(\breve{\psi})],[I^{L-}_{\breve{Z}}(\breve{\psi}),I^{R-}_{\breve{Z}}(\breve{\psi})],[F^{L-}_{\breve{Z}}(\breve{\psi}),F^{R-}_{\breve{Z}}(\breve{\psi})]\end{array}\right\rangle:\breve{\psi}\in\breve{X}\right\}$$

where $\breve{s}_{\partial_{\breve{Z}}(\breve{\psi})}$ is the linguistic variable, $T^{L+}_{\breve{Z}},T^{R+}_{\breve{Z}},I^{L+}_{\breve{Z}},I^{R+}_{\breve{Z}},F^{L+}_{\breve{Z}},F^{R+}_{\breve{Z}}:\breve{X}\to[0,1]$ and $T^{L-}_{\breve{Z}},T^{R-}_{\breve{Z}},I^{L-}_{\breve{Z}},I^{R-}_{\breve{Z}},F^{L-}_{\breve{Z}},F^{R-}_{\breve{Z}}:\breve{X}\to[-1,0]$. $T^{L+}_{\breve{Z}},T^{R+}_{\breve{Z}},I^{L+}_{\breve{Z}},I^{R+}_{\breve{Z}},F^{L+}_{\breve{Z}},F^{R+}_{\breve{Z}}$ and $T^{L-}_{\breve{Z}},T^{R-}_{\breve{Z}},I^{L-}_{\breve{Z}},I^{R-}_{\breve{Z}},F^{L-}_{\breve{Z}},F^{R-}_{\breve{Z}}$ respectively denote the positive truth, indeterminacy, and falsity membership degrees of the IBL_NS $\breve{Z}$.

Definition 2: Operational rules of IBL_NS.

Let $\breve{Z}_1=\{\langle\breve{\psi},\breve{s}_{\partial(\breve{Z}_1)},[T^{L+}_{\breve{Z}_1}(\breve{\psi}),T^{R+}_{\breve{Z}_1}(\breve{\psi})],[I^{L+}_{\breve{Z}_1}(\breve{\psi}),I^{R+}_{\breve{Z}_1}(\breve{\psi})],[F^{L+}_{\breve{Z}_1}(\breve{\psi}),F^{R+}_{\breve{Z}_1}(\breve{\psi})],[T^{L-}_{\breve{Z}_1}(\breve{\psi}),T^{R-}_{\breve{Z}_1}(\breve{\psi})],[I^{L-}_{\breve{Z}_1}(\breve{\psi}),I^{R-}_{\breve{Z}_1}(\breve{\psi})],[F^{L-}_{\breve{Z}_1}(\breve{\psi}),F^{R-}_{\breve{Z}_1}(\breve{\psi})]\rangle:\breve{\psi}\in\breve{X}\}$ and

$\breve{Z}_2=\{\langle\breve{\psi},\breve{s}_{\partial(\breve{Z}_2)},[T^{L+}_{\breve{Z}_2}(\breve{\psi}),T^{R+}_{\breve{Z}_2}(\breve{\psi})],[I^{L+}_{\breve{Z}_2}(\breve{\psi}),I^{R+}_{\breve{Z}_2}(\breve{\psi})],[F^{L+}_{\breve{Z}_2}(\breve{\psi}),F^{R+}_{\breve{Z}_2}(\breve{\psi})],[T^{L-}_{\breve{Z}_2}(\breve{\psi}),T^{R-}_{\breve{Z}_2}(\breve{\psi})],[I^{L-}_{\breve{Z}_2}(\breve{\psi}),I^{R-}_{\breve{Z}_2}(\breve{\psi})],[F^{L-}_{\breve{Z}_2}(\breve{\psi}),F^{R-}_{\breve{Z}_2}(\breve{\psi})]\rangle:\breve{\psi}\in\breve{X}\}$

be two IBL_NSs and $\partial\geq 0$, then the operational rules of IBL_NSs are defined in Eq. (1).

$$\breve{Z}_1+\breve{Z}_2=\left\langle\begin{array}{l}[\breve{s}_{\partial(\breve{Z}_1)+\theta(\breve{Z}_2)}],\\ \left[\begin{array}{l}T^{L+}_{\breve{Z}_1}(\breve{\psi})+T^{L+}_{\breve{Z}_2}(\breve{\psi})-T^{L+}_{\breve{Z}_1}(\breve{\psi})T^{L+}_{\breve{Z}_2}(\breve{\psi}),T^{R+}_{\breve{Z}_1}(\breve{\psi})\\ +T^{R+}_{\breve{Z}_2}(\breve{\psi})-T^{R+}_{\breve{Z}_1}(\breve{\psi})T^{R+}_{\breve{Z}_2}(\breve{\psi})\end{array}\right],\\ \left[I^{L+}_{\breve{Z}_1}(\breve{\psi})I^{L+}_{\breve{Z}_2}(\breve{\psi}),I^{R+}_{\breve{Z}_1}(\breve{\psi})I^{R+}_{\breve{Z}_2}(\breve{\psi})\right],\\ \left[F^{L+}_{\breve{Z}_1}(\breve{\psi})F^{L+}_{\breve{Z}_2}(\breve{\psi}),F^{R+}_{\breve{Z}_1}(\breve{\psi})F^{R+}_{\breve{Z}_2}(\breve{\psi})\right],\\ \left[-T^{L-}_{\breve{Z}_1}(\breve{\psi})T^{L-}_{\breve{Z}_2}(\breve{\psi}),-T^{R-}_{\breve{Z}_1}(\breve{\psi})T^{R-}_{\breve{Z}_2}(\breve{\psi})\right],\\ \left[\begin{array}{l}-(-I^{L-}_{\breve{Z}_1}(\breve{\psi})-I^{L-}_{\breve{Z}_2}(\breve{\psi})-I^{L-}_{\breve{Z}_1}(\breve{\psi}).I^{L-}_{\breve{Z}_2}(\breve{\psi})),\\ -(-I^{R-}_{\breve{Z}_1}(\breve{\psi})-I^{R-}_{\breve{Z}_2}(\breve{\psi})-I^{R-}_{\breve{Z}_1}(\breve{\psi}).I^{R-}_{\breve{Z}_2}(\breve{\psi}))\end{array}\right],\\ \left[\begin{array}{l}-(-F^{L-}_{\breve{Z}_1}(\breve{\psi})-F^{L-}_{\breve{Z}_2}(\breve{\psi})-F^{L-}_{\breve{Z}_1}(\breve{\psi})F^{L-}_{\breve{Z}_2}(\breve{\psi})),\\ -(-F^{R-}_{\breve{Z}_1}(\breve{\psi})-F^{R-}_{\breve{Z}_2}(\breve{\psi})-F^{R-}_{\breve{Z}_1}(\breve{\psi})F^{R-}_{\breve{Z}_2}(\breve{\psi}))\end{array}\right]\end{array}\right\rangle \quad (1)$$

$$\breve{Z}_1.\breve{Z}_2=\left\langle\begin{array}{l}[\breve{s}_{\partial(\breve{Z}_1)x\theta(\breve{Z}_2)}],\left[T^{L+}_{\breve{Z}_1}(\breve{\psi})T^{L+}_{\breve{Z}_2}(\breve{\psi}),T^{R+}_{\breve{Z}_1}(\breve{\psi})T^{R+}_{\breve{Z}_2}(\breve{\psi})\right],\\ \left[\begin{array}{l}I^{L+}_{\breve{Z}_1}(\breve{\psi})+I^{L+}_{\breve{Z}_2}(\breve{\psi})-I^{L+}_{\breve{Z}_1}(\breve{\psi})I^{L+}_{\breve{Z}_2}(\breve{\psi}),I^{R+}_{\breve{Z}_1}(\breve{\psi})\\ +I^{R+}_{\breve{Z}_2}(\breve{\psi})-I^{R+}_{\breve{Z}_1}(\breve{\psi})I^{R+}_{\breve{Z}_2}(\breve{\psi})\end{array}\right],\\ \left[\begin{array}{l}F^{L+}_{\breve{Z}_1}(\breve{\psi})+F^{L+}_{\breve{Z}_2}(\breve{\psi})-F^{L+}_{\breve{Z}_1}(\breve{\psi})F^{L+}_{\breve{Z}_2}(\breve{\psi}),F^{R+}_{\breve{Z}_1}(\breve{\psi})\\ +F^{R+}_{\breve{Z}_2}(\breve{\psi})-F^{R+}_{\breve{Z}_1}(\breve{\psi})F^{R+}_{\breve{Z}_2}(\breve{\psi})\end{array}\right],\\ \left[\begin{array}{l}-(-T^{L-}_{\breve{Z}_1}(\breve{\psi})-T^{L-}_{\breve{Z}_2}(\breve{\psi})-T^{L-}_{\breve{Z}_1}(\breve{\psi})T^{L-}_{\breve{Z}_2}(\breve{\psi})),\\ -(-T^{R-}_{\breve{Z}_1}(\breve{\psi})-T^{R-}_{\breve{Z}_2}(\breve{\psi})-T^{R-}_{\breve{Z}_1}(\breve{\psi})T^{R-}_{\breve{Z}_2}(\breve{\psi}))\end{array}\right],\\ \left[-I^{L-}_{\breve{Z}_1}(\breve{\psi}).I^{L-}_{\breve{Z}_2}(\breve{\psi}),-I^{R-}_{\breve{Z}_1}(\breve{\psi}).I^{R-}_{\breve{Z}_2}(\breve{\psi})\right],\\ \left[-F^{L-}_{\breve{Z}_1}(\breve{\psi})F^{L-}_{\breve{Z}_2}(\breve{\psi}),-F^{R-}_{\breve{Z}_1}(\breve{\psi})F^{R-}_{\breve{Z}_2}(\breve{\psi})\right]\end{array}\right\rangle \quad (2)$$

$$\lambda.\breve{Z}_1=\left\langle\begin{array}{l}\breve{s}_{\lambda.\partial(\breve{Z}_1)},[1-(1-T^{L+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda},1-(1-T^{R+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}],\\ [(I^{L+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda},(I^{R+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}],[(F^{L+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda},(F^{R+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}],\\ [-(-T^{L-}_{\breve{Z}_1}(\breve{\psi}))^{\lambda},-(-T^{R-}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}],[-(-I^{L-}_{\breve{Z}_1}(\breve{\psi}))^{\lambda},-(-I^{R-}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}],\\ [-(1-(1-(-F^{L-}_{\breve{Z}_1}(\breve{\psi})))^{\lambda}),-(1-(1-(-F^{R-}_{\breve{Z}_1}(\breve{\psi})))^{\lambda})]\end{array}\right\rangle \quad (3)$$

$$\left(\breve{Z}_1(\breve{\psi})\right)^{\lambda}=\left\langle\begin{array}{l}[\breve{s}_{\partial^{\lambda}(\breve{Z}_1)}],\left[(T^{L+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda},(T^{R+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}\right],\\ \left[1-(1-I^{L+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda},1-(1-I^{R+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}\right],\\ \left[1-(1-F^{L+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda},1-(1-F^{R+}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}\right],\\ \left[-(1-(1-(-T^{L-}_{\breve{Z}_1}(\breve{\psi})))^{\lambda}),-(1-(1-(-T^{R-}_{\breve{Z}_1}(\breve{\psi})))^{\lambda})\right],\\ \left[-(1-(1-(-I^{L-}_{\breve{Z}_1}(\breve{\psi})))^{\lambda}),-(1-(1-(-I^{R-}_{\breve{Z}_1}(\breve{\psi})))^{\lambda})\right],\\ \left[-(-F^{L-}_{\breve{Z}_1}(\breve{\psi}))^{\lambda}-(-F^{R-}_{\breve{Z}_1}(\breve{\psi}))^{\lambda})\right]\end{array}\right\rangle \quad (4)$$

Definition 3. Union and intersection of the IBL_NSs.

Let $\breve{Z}_1$ and $\breve{Z}_2$ be two IBL_NSs over $\breve{X}$ which are defined by:

$$\breve{Z}_1=\begin{array}{l}\{\langle\breve{\psi},\breve{s}_{\partial(\breve{Z}_1)},[T^{L+}_{\breve{Z}_1}(\breve{\psi}),T^{R+}_{\breve{Z}_1}(\breve{\psi})],[I^{L+}_{\breve{Z}_1}(\breve{\psi}),I^{R+}_{\breve{Z}_1}(\breve{\psi})],[F^{L+}_{\breve{Z}_1}(\breve{\psi}),F^{R+}_{\breve{Z}_1}(\breve{\psi})],\\ [T^{L-}_{\breve{Z}_1}(\breve{\psi}),T^{R-}_{\breve{Z}_1}(\breve{\psi})],[I^{L-}_{\breve{Z}_1}(\breve{\psi}),I^{R-}_{\breve{Z}_1}(\breve{\psi})],[F^{L-}_{\breve{Z}_1}(\breve{\psi}),F^{R-}_{\breve{Z}_1}(\breve{\psi})]\rangle:\breve{\psi}\in\breve{X}\}\end{array}$$

and

$$\breve{Z}_2=\begin{array}{l}\{\langle\breve{\psi},\breve{s}_{\partial(\breve{Z}_2)},[T^{L+}_{\breve{Z}_2}(\breve{\psi}),T^{R+}_{\breve{Z}_2}(\breve{\psi})],[I^{L+}_{\breve{Z}_2}(\breve{\psi}),I^{R+}_{\breve{Z}_2}(\breve{\psi})],[F^{L+}_{\breve{Z}_2}(\breve{\psi}),F^{R+}_{\breve{Z}_2}(\breve{\psi})],\\ [T^{L-}_{\breve{Z}_2}(\breve{\psi}),T^{R-}_{\breve{Z}_2}(\breve{\psi})],[I^{L-}_{\breve{Z}_2}(\breve{\psi}),I^{R-}_{\breve{Z}_2}(\breve{\psi})],[F^{L-}_{\breve{Z}_2}(\breve{\psi}),F^{R-}_{\breve{Z}_2}(\breve{\psi})]\rangle:\breve{\psi}\in\breve{X}\}\end{array}$$

respectively. Their *union* denoted as $\breve{Z}_1\cup\breve{Z}_2$ and is defined in Eq. (5):

$$(\breve{Z}_1 \cup \breve{Z}_2)(\breve{\psi}) = \breve{s}_{\partial_{\breve{Z}_1\cup \breve{Z}_2}(\psi)} = \vee(s_{\partial_{\breve{Z}_1}(\psi)}, s_{\partial_{\breve{Z}_2}(\psi)}),$$
$$\left\{\begin{array}{l} T^{+}_{\breve{Z}_1\cup\breve{Z}_2}(\psi) = \vee([T^{+L}_{\breve{Z}_1}(\psi), T^{+R}_{\breve{Z}_1}(\psi)],[T^{+L}_{\breve{Z}_2}(\psi), T^{+R}_{\breve{Z}_2}(\psi)]); \\ \qquad \vee([T^{-L}_{\breve{Z}_1}(\psi), T^{-R}_{\breve{Z}_1}(\psi)],[T^{-L}_{\breve{Z}_2}(\psi), T^{-R}_{\breve{Z}_2}(\psi)]), \\ I^{+}_{\breve{Z}_1\cup\breve{Z}_2}(\breve{\psi}) = \wedge([I^{+L}_{\breve{Z}_1}(\breve{\psi}), I^{+R}_{\breve{Z}_1}(\breve{\psi})],[I^{+L}_{\breve{Z}_2}(\breve{\psi}), I^{+R}_{\breve{Z}_2}(\breve{\psi})]); \\ \qquad \wedge([I^{-L}_{\breve{Z}_1}(\breve{\psi}), I^{-R}_{\breve{Z}_1}(\breve{\psi})],[I^{-L}_{\breve{Z}_2}(\breve{\psi}), I^{-R}_{\breve{Z}_2}(\breve{\psi})]), \\ F^{+}_{\breve{Z}_1\cup\breve{Z}_2}(\psi) = \wedge([F^{+L}_{\breve{Z}_1}(\psi), F^{+R}_{\breve{Z}_1}(\psi)],[F^{+L}_{\breve{Z}_2}(\psi), F^{+R}_{\breve{Z}_2}(\psi)]); \\ \qquad \wedge([F^{-L}_{\breve{Z}_1}(\psi), F^{-R}_{\breve{Z}_1}(\psi)],[F^{-L}_{\breve{Z}_2}(\psi), F^{-R}_{\breve{Z}_2}(\psi)]). \end{array}\right\} \quad (5)$$

for all $\breve{\psi} \in \breve{X}$. The symbols $\vee, \wedge$ represents max and min operators.

Definition 4. Let $\breve{Z}_1$ and $\breve{Z}_2$ be two IBL_NSs over X which are defined by:

$$Z_1 = \{\langle \psi, s_{\partial_{z_1}(\breve{\psi})}, ([T^{+L}_{\breve{Z}_1}(\psi), T^{+R}_{\breve{Z}_1}(\psi)],[I^{+L}_{\breve{Z}_1}(\psi), I^{+R}_{\breve{Z}_1}(\psi)],[F^{+L}_{\breve{Z}_1}(\psi), F^{+R}_{\breve{Z}_1}(\psi)], [T^{-L}_{\breve{Z}_1}(\psi), T^{-R}_{\breve{Z}_1}(\psi)],[I^{-L}_{\breve{Z}_1}(\psi), I^{-R}_{\breve{Z}_1}(\psi)],[F^{-L}_{\breve{Z}_1}(\psi), F^{-R}_{\breve{Z}_1}(\psi)])\rangle \mid \psi \in X\},$$

and

$$Z_2 = \{\langle \breve{\psi}, \breve{s}_{\partial_{\breve{Z}_2}(\breve{\psi})}, ([T^{+L}_{\breve{Z}_2}(\breve{\psi}), T^{+R}_{\breve{Z}_2}(\breve{\psi})],[I^{+L}_{\breve{Z}_2}(\breve{\psi}), I^{+R}_{\breve{Z}_2}(\breve{\psi})],[F^{+L}_{\breve{Z}_2}(\breve{\psi}), F^{+R}_{\breve{Z}_2}(\breve{\psi})], [T^{-L}_{\breve{Z}_2}(\psi), T^{-R}_{\breve{Z}_2}(\psi)],[I^{-L}_{\breve{Z}_2}(\psi), I^{-R}_{\breve{Z}_2}(\psi)],[F^{-L}_{\breve{Z}_2}(\psi), F^{-R}_{\breve{Z}_2}(\psi)])\rangle \mid \psi \in X\},$$

respectively. Their *intersection* denoted as $\breve{Z}_1 \cap \breve{Z}_2$ and is defined in Eq. (6):

$$(\breve{Z}_1 \cap \breve{Z}_2)(\breve{\psi}) = \breve{s}_{\partial_{z_1\cap z_2}(\breve{\psi})} = \wedge(\breve{s}_{\partial_{Z_1}(\breve{\psi})}, \breve{s}_{\partial_{Z_2}(\breve{\psi})}),$$
$$\left\{\begin{array}{l} T^{+}_{\breve{Z}_1\cap\breve{Z}_2}(\psi) = \wedge([T^{+L}_{\breve{Z}_1}(\psi), T^{+R}_{\breve{Z}_1}(\psi)],[T^{+L}_{\breve{Z}_2}(\psi), T^{+R}_{\breve{Z}_2}(\psi)]); \\ \qquad \wedge([T^{-L}_{\breve{Z}_1}(\psi), T^{-R}_{\breve{Z}_1}(\psi)],[T^{-L}_{\breve{Z}_2}(\psi), T^{-R}_{\breve{Z}_2}(\psi)]), \\ I^{+}_{Z_1\cap Z_2}(\psi) = \vee([I^{+L}_{\breve{Z}_1}(\psi), I^{+R}_{\breve{Z}_1}(\psi)],[I^{+L}_{\breve{Z}_2}(\psi), I^{+R}_{\breve{Z}_2}(\psi)]); \\ \qquad \vee([I^{-L}_{\breve{Z}_1}(\breve{\psi}), I^{-R}_{\breve{Z}_1}(\breve{\psi})],[I^{-L}_{\breve{Z}_2}(\breve{\psi}), I^{-R}_{\breve{Z}_2}(\breve{\psi})]), \\ F^{+}_{\breve{Z}_1\cap\breve{Z}_2}(\psi) = \vee([F^{+L}_{\breve{Z}_1}(\psi), F^{+R}_{\breve{Z}_1}(\psi)],[F^{+L}_{\breve{Z}_2}(\psi), F^{+R}_{\breve{Z}_2}(\psi)]); \\ \qquad \vee([F^{-L}_{\breve{Z}_1}(\psi), F^{-R}_{\breve{Z}_1}(\psi)],[F^{-L}_{\breve{Z}_2}(\psi), F^{-R}_{\breve{Z}_2}(\psi)]) \end{array}\right\} \quad (6)$$

for all $\breve{\psi} \in \breve{X}$. The symbols $\vee, \wedge$ represents max and min operators.

Proposition 1. Let $\breve{\theta}_1$ and $\breve{\theta}_2$ be two IBL_NSs over $\breve{X}$. Then

- $\breve{\theta}_1 \cup \breve{\theta}_2 = \breve{\theta}_2 \cup \breve{\theta}_1,$
- $\breve{\theta}_1 \cap \breve{\theta}_2 = \breve{\theta}_2 \cap \breve{\theta}_1,$
- $\breve{\theta}_1 \cup \breve{\theta}_1 = \breve{\theta}_1,$
- $\breve{\theta}_1 \cap \breve{\theta}_1 = \breve{\theta}_1.$

Proposition 2. Let $\breve{\theta}_1, \breve{\theta}_2$ and $\breve{\theta}_3$ be three IBL_NSs over $\breve{X}$. Then,

- $\breve{\theta}_1 \cup (\breve{\theta}_2 \cup \breve{\theta}_3) = (\breve{\theta}_1 \cup \breve{\theta}_2) \cup \breve{\theta}_3,$
- $\breve{\theta}_1 \cap (\breve{\theta}_2 \cap \breve{\theta}_3) = (\breve{\theta}_1 \cap \breve{\theta}_2) \cap \breve{\theta}_3,$

- $\breve{\theta}_1 \cup (\breve{\theta}_2 \cap \breve{\theta}_3) = (\breve{\theta}_1 \cup \breve{\theta}_2) \cap (\breve{\theta}_1 \cup \breve{\theta}_3),$
- $\breve{\theta}_1 \cap (\breve{\theta}_2 \cup \breve{\theta}_3) = (\breve{\theta}_1 \cap \breve{\theta}_2) \cup (\breve{\theta}_1 \cap \breve{\theta}_3),$
- $\breve{\theta}_1 \cup (\breve{\theta}_1 \cap \breve{\theta}_2) = \breve{\theta}_1,$
- $\breve{\theta}_1 \cap (\breve{\theta}_1 \cup \breve{\theta}_2) = \breve{\theta}_1.$

Definition 5. Let $\breve{Z}_1$ and $\breve{Z}_2$ be two IBL_NSs over $\breve{X}$:

$$\breve{Z}_1 = \left\{ \begin{array}{l} \langle \psi, s_{\partial_{\breve{Z}_1}(\psi)}, ([T^{+L}_{\breve{Z}_1}(\psi), T^{+R}_{\breve{Z}_1}(\psi)], [I^{+L}_{\breve{Z}_1}(\psi), I^{+R}_{\breve{Z}_1}(\psi)], [F^{+L}_{\breve{Z}_1}(\psi), F^{+R}_{\breve{Z}_1}(\psi)], \\ [T^{-L}_{\breve{Z}_1}(\psi), T^{-R}_{\breve{Z}_1}(\psi)], [I^{-L}_{\breve{Z}_1}(\psi), I^{-R}_{\breve{Z}_1}(\psi)], [F^{-L}_{\breve{Z}_1}(\psi), F^{-R}_{\breve{Z}_1}(\psi)]) \rangle \mid \psi \in \breve{X} \end{array} \right\}$$

and

$$\breve{Z}_2 = \left\{ \begin{array}{l} \langle \psi, s_{\partial_{\breve{Z}_2}(\psi)}, ([T^{+L}_{\breve{Z}_2}(\psi), T^{+R}_{\breve{Z}_2}(\psi)], [I^{+L}_{\breve{Z}_2}(\psi), I^{+R}_{\breve{Z}_2}(\psi)], [F^{+L}_{\breve{Z}_2}(\psi), F^{+R}_{\breve{Z}_2}(\psi)], \\ [T^{-L}_{\breve{Z}_2}(\psi), T^{-R}_{\breve{Z}_2}(\psi)], [I^{-L}_{\breve{Z}_2}(\psi), I^{-R}_{\breve{Z}_2}(\psi)], [F^{-L}_{\breve{Z}_2}(\psi), F^{-R}_{\breve{Z}_2}(\psi)]) \rangle \mid \psi \in \breve{X} \end{array} \right\}$$

Then,

(1) The Hamming distance between $\breve{Z}_1$ and $\breve{Z}_2$ is defined in Eq. (7):

$$\breve{d}_H(\breve{Z}_1, \breve{Z}_2) = \frac{1}{12(n-1)} \left(\begin{array}{l} |\partial_{\breve{Z}_1} \times T^{+L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times T^{+L}_{\breve{Z}_2}| + |\partial_{\breve{Z}_1} \times T^{+R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times T^{+R}_{\breve{Z}_2}| \\ + |\partial_{\breve{Z}_1} \times I^{+L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times I^{+L}_{\breve{Z}_2}| + |\partial_{\breve{Z}_1} \times I^{+R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times I^{+R}_{\breve{Z}_2}| \\ + |\partial_{\breve{Z}_1} \times F^{+L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times F^{+L}_{\breve{Z}_2}| + |\partial_{\breve{Z}_1} \times F^{+R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times F^{+R}_{\breve{Z}_2}| \\ + |\partial_{\breve{Z}_1} \times T^{-L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times T^{-L}_{\breve{Z}_2}| + |\partial_{\breve{Z}_1} \times T^{-R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times T^{-R}_{\breve{Z}_2}| \\ + |\partial_{\breve{Z}_1} \times I^{-L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times I^{-L}_{\breve{Z}_2}| + |\partial_{\breve{Z}_1} \times I^{-R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times I^{-R}_{\breve{Z}_2}| \\ + |\partial_{\breve{Z}_1} \times F^{-L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times F^{+L}_{\breve{Z}_2}| + |\partial_{\breve{Z}_1} \times F^{-R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times F^{-R}_{\breve{Z}_2}| \end{array} \right) \tag{7}$$

(2) The Euclidian distance between $\breve{Z}_1$ and $\breve{Z}_2$ is defined in Eq. (8):

$$\breve{d}_E(\breve{Z}_1, \breve{Z}_2) = \sqrt{\frac{1}{12(n-1)} \left(\begin{array}{l} (\partial_{\breve{Z}_1} \times T^{+L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times T^{+L}_{\breve{Z}_2})^2 + (\partial_{\breve{Z}_1} \times T^{+R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times T^{+R}_{\breve{Z}_2})^2 \\ +(\partial_{\breve{Z}_1} \times I^{+L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times I^{+L}_{\breve{Z}_2})^2 + (\partial_{\breve{Z}_1} \times I^{+R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times I^{+R}_{\breve{Z}_2})^2 \\ +(\partial_{\breve{Z}_1} \times F^{+L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times F^{+L}_{\breve{Z}_2})^2 + (\partial_{\breve{Z}_1} \times F^{+R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times F^{+R}_{\breve{Z}_2})^2 \\ +(\partial_{\breve{Z}_1} \times T^{-L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times T^{-L}_{\breve{Z}_2})^2 + (\partial_{\breve{Z}_1} \times T^{-R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times T^{-R}_{\breve{Z}_2})^2 \\ +(\partial_{\breve{Z}_1} \times I^{-L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times I^{-L}_{\breve{Z}_2})^2 + (\partial_{\breve{Z}_1} \times I^{-R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times I^{-R}_{\breve{Z}_2})^2 \\ +(\partial_{\breve{Z}_1} \times F^{-L}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times F^{-L}_{\breve{Z}_2})^2 + (\partial_{\breve{Z}_1} \times F^{-R}_{\breve{Z}_1} - \partial_{\breve{Z}_2} \times F^{-R}_{\breve{Z}_2})^2 \end{array} \right)} \tag{8}$$

4. The extended TOPSIS method based on IBL_NSs

This section develops a new TOPSIS method using IBL_NS. A committee of k decision makers $(\breve{D}_t, t = 1 \dots h)$ is assumed responsible for evaluating m alternatives $(\breve{A}_i, i = 1 \dots m)$ under n criteria $(\breve{C}_j, j = 1, \dots, n),$ where the ratings of alternatives and the importance weights of criteria are expressed by using IBL_NS. The steps of the proposed IBL_NS-TOPSIS approach are as follows:

Step 1. Aggregate the ratings of alternatives.

Let $\breve{x}_{ijt} = \left\langle \breve{\psi}, \left\{ \breve{s}_{\partial_{ijt}}(\breve{\psi}), \left(\begin{array}{l} [T_{ijt}^{+L}(\breve{\psi}), T_{ijt}^{+R}(\breve{\psi})], [I_{ijt}^{+L}(\breve{\psi}), I_{ijt}^{+R}(\breve{\psi})], [F_{ijt}^{+L}(\breve{\psi}), F_{ijt}^{+R}(\breve{\psi})], \\ [T_{ijt}^{-L}(\breve{\psi}), T_{ijt}^{-R}(\breve{\psi})], [I_{ijt}^{-L}(\breve{\psi}), I_{ijt}^{-R}(\breve{\psi})], [F_{ijt}^{-L}(\breve{\psi}), F_{ijt}^{-R}(\breve{\psi})] \end{array} \right) \right\} \right\rangle$ be the suitability ratings assigned to $\breve{Y}_i$ by $\breve{D}_t$ for $\breve{C}_j$.

where: $\breve{s}_{\partial_{ijt}(\breve{\psi})}$ are the linguistic variables, $T_{ijt}^{+L}, T_{ijt}^{+R}, I_{ijt}^{+L}, I_{ijt}^{+R}, F_{ijt}^{+L}, F_{ijt}^{+R} : \breve{X} \rightarrow [0,1]$ and $T_{ijt}^{-L}, T_{ijt}^{-L}, I_{ijt}^{-L}, I_{ijt}^{-L}, F_{ijt}^{-L}, F_{ijt}^{-L} : \breve{X} \rightarrow [-1,0]$ with the condition $0 \le T_{ijR}^{+}(\breve{\psi}), T_{ijR}^{-}(\breve{\psi}), I_{ijR}^{+}(\breve{\psi}), \; I_{ijR}^{-}(\breve{\psi}), F_{ijR}^{+}(\breve{\psi}), F_{ijR}^{-}(\breve{\psi}) \le 6$ for any $\breve{\psi} \in \breve{X}$.

The average ratings $\breve{x}_{ij} = \left\langle \breve{\psi}, \left\{ \breve{s}_{\partial_{ij}}(\breve{\psi}), \left(\begin{array}{l} [T_{ij}^{+L}(\breve{\psi}), T_{ij}^{+R}(\breve{\psi})], [I_{ij}^{+L}(\breve{\psi}), I_{ij}^{+R}(\breve{\psi})], [F_{ij}^{+L}(\breve{\psi}), F_{ij}^{+R}(\breve{\psi})], \\ [T_{ij}^{-L}(\breve{\psi}), T_{ij}^{-R}(\breve{\psi})], [I_{ij}^{-L}(\breve{\psi}), I_{ij}^{-R}(\breve{\psi})], [F_{ij}^{-L}(\breve{\psi}), F_{ij}^{-R}(\breve{\psi})] \end{array} \right) \right\} \right\rangle$ are calculated using Eq. (9).

$$\breve{x}_{ij} = \frac{1}{h} \otimes (\breve{x}_{ij1} \oplus \breve{x}_{ij2} \oplus \ldots \oplus \breve{x}_{ijt}), \tag{9}$$

where:

$$T_{ij}^{+}(\breve{\psi}) = \left[1 - \left(1 - \sum_{t=1}^{h} T_{ijt}^{+L}(\breve{\psi}) \right)^{\frac{1}{h}}, 1 - \left(1 - \sum_{t=1}^{h} T_{ijt}^{+R}(\breve{\psi}) \right)^{\frac{1}{h}} \right],$$

$$T_{ij}^{-}(\breve{\psi}) = \left[1 - \left(1 - \sum_{t=1}^{h} T_{ijt}^{-L}(\breve{\psi}) \right)^{\frac{1}{h}}, 1 - \left(1 - \sum_{t=1}^{h} T_{ijt}^{-R}(\breve{\psi}) \right)^{\frac{1}{h}} \right],$$

$$I_{ij}^{+}(\breve{\psi}) = \left[\left(\sum_{t=1}^{h} I_{ijt}^{+L}(\breve{\psi}) \right)^{\frac{1}{h}}, \left(\sum_{t=1}^{h} T_{ijt}^{+R}(\breve{\psi}) \right)^{\frac{1}{h}} \right],$$

$$I_{ij}^{-}(\breve{\psi}) = \left[\left(\sum_{t=1}^{h} I_{ijt}^{-L}(\breve{\psi}) \right)^{\frac{1}{h}}, \left(\sum_{t=1}^{h} T_{ijt}^{-R}(\breve{\psi}) \right)^{\frac{1}{h}} \right],$$

$$F_{ij}^{+}(\breve{\psi}) = \left[\left(\sum_{t=1}^{h} F_{ijt}^{+L}(\breve{\psi}) \right)^{\frac{1}{h}}, \left(\sum_{t=1}^{h} F_{ijt}^{+R}(\breve{\psi}) \right)^{\frac{1}{h}} \right],$$

$$F_{ij}^{-}(\breve{\psi}) = \left[\left(\sum_{t=1}^{h} F_{ijt}^{-L}(\breve{\psi}) \right)^{\frac{1}{h}}, \left(\sum_{t=1}^{h} F_{ijt}^{-R}(\breve{\psi}) \right)^{\frac{1}{h}} \right].$$

Step 2. Aggregate the importance weights.

Let $\breve{W}_{jt} = \left\langle \breve{\psi}, \left\{ \breve{s}_{\chi_{jt}}(\breve{\psi}), \left(\begin{array}{l} [T_{jt}^{+L}(\breve{\psi}), T_{jt}^{+R}(\breve{\psi})], [I_{jt}^{+L}(\breve{\psi}), I_{jt}^{+R}(\breve{\psi})], [F_{jt}^{+L}(\breve{\psi}), F_{jt}^{+R}(\breve{\psi})], \\ [T_{jt}^{-L}(\breve{\psi}), T_{jt}^{-R}(\breve{\psi})], [I_{jt}^{-L}(\breve{\psi}), I_{jt}^{-R}(\breve{\psi})], [F_{jt}^{-L}(\breve{\psi}), F_{jt}^{-R}(\breve{\psi})] \end{array} \right) \right\} \right\rangle$ be the importance weights assigned by $\breve{D}_t$ to $\breve{C}_j$. The average importance weights of criteria $\breve{W}_j = \left\langle \breve{\psi}, \left\{ \breve{s}_{\chi_j}(\breve{\psi}), \left(\begin{array}{l} [T_{j}^{+L}(\breve{\psi}), T_{j}^{+R}(\breve{\psi})], [I_{j}^{+L}(\breve{\psi}), I_{j}^{+R}(\breve{\psi})], [F_{j}^{+L}(\breve{\psi}), F_{j}^{+R}(\breve{\psi})], \\ [T_{j}^{-L}(\breve{\psi}), T_{j}^{-R}(\breve{\psi})], [I_{j}^{-L}(\breve{\psi}), I_{j}^{-R}(\breve{\psi})], [F_{j}^{-L}(\breve{\psi}), F_{j}^{-R}(\breve{\psi})] \end{array} \right) \right\} \right\rangle$ can be evaluated as:

$$\breve{w}_{j} = \frac{1}{h} \otimes (\breve{w}_{j1} \oplus \breve{w}_{j2} \oplus \ldots \oplus \breve{w}_{jt}), \tag{10}$$

where:

$$T_j^{+}(\breve{\psi})=\left[1-\left(1-\sum_{t=1}^{h}T_{jt}^{+L}(\breve{\psi})\right)^{\frac{1}{h}},1-\left(1-\sum_{t=1}^{h}T_{jt}^{+R}(\breve{\psi})\right)^{\frac{1}{h}}\right],$$

$$T_j^{-}(\breve{\psi})=\left[1-\left(1-\sum_{t=1}^{h}T_{jt}^{-L}(\breve{\psi})\right)^{\frac{1}{h}},1-\left(1-\sum_{t=1}^{h}T_{jt}^{-R}(\breve{\psi})\right)^{\frac{1}{h}}\right]$$

$$I_j^{+}(\psi)=\left[\left(\sum_{t=1}^{h}I_{jt}^{+L}(\psi)\right)^{\frac{1}{h}},\left(\sum_{t=1}^{h}T_{jt}^{+R}(\psi)\right)^{\frac{1}{h}}\right],$$

$$I_j^{-}(\psi)=\left[\left(\sum_{t=1}^{h}I_{jt}^{-L}(\psi)\right)^{\frac{1}{h}},\left(\sum_{t=1}^{h}T_{jt}^{-R}(\psi)\right)^{\frac{1}{h}}\right]$$

$$F_j^{+}(\psi)=\left[\left(\sum_{t=1}^{h}F_{jt}^{+L}(\psi)\right)^{\frac{1}{h}},\left(\sum_{t=1}^{h}F_{jt}^{+R}(\psi)\right)^{\frac{1}{h}}\right],$$

$$F_j^{-}(\psi)=\left[\left(\sum_{t=1}^{h}F_{jt}^{-L}(\psi)\right)^{\frac{1}{h}},\left(\sum_{t=1}^{h}F_{jt}^{-R}(\psi)\right)^{\frac{1}{h}}\right]$$

Step 3. Aggregate the weighted ratings of alternatives.

The weighted ratings of alternatives $\breve{Y}_i$ can be defined as in Eq. (11):

$$\breve{Y}_i=\frac{1}{n}\sum_{j=1}^{n}\breve{x}_{ij}\otimes\breve{w}_j, i=1,\ldots,m;\ j=1,\ldots,n.$$

$$\breve{Y}_i=\left\{\breve{s}_{\upsilon_i}(\psi),\left(\begin{array}{l}[T_i^{+L}(\psi),T_i^{+R}(\psi)],[I_i^{+L}(\psi),I_i^{+R}(\psi)],[F_i^{+L}(\psi),F_i^{+R}(\psi)],\\ [T_i^{-L}(\psi),T_i^{-R}(\psi)],[I_i^{-L}(\psi),I_i^{-R}(\psi)],[F_i^{-L}(\psi),F_i^{-R}(\psi)]\end{array}\right)\right\}, \tag{11}$$

where:

$$\breve{s}_{\upsilon_i}(\breve{\psi})=\breve{s}_{\partial_{ij}}(\breve{\psi})\,\mathrm{x}\,\breve{s}_{\chi_j}(\breve{\psi})$$

$$T_i^{+L}(\psi)=1-\left\{1-\left(1-\left(1-\sum_{t=1}^{h}T_{ijt}^{+L}(\psi)\right)^{\frac{1}{h}}\right)*\left(1-\left(1-\sum_{t=1}^{h}T_{jt}^{+L}(\psi)\right)^{\frac{1}{h}}\right)\right\}^{\frac{1}{n}}$$

$$T_i^{+R}(\psi)=1-\left\{1-\left(1-\left(1-\sum_{t=1}^{h}T_{ijt}^{+R}(\psi)\right)^{\frac{1}{h}}\right)*\left(1-\left(1-\sum_{t=1}^{h}T_{jt}^{+R}(\psi)\right)^{\frac{1}{h}}\right)\right\}^{\frac{1}{n}}$$

$$I_i^{+L}(\psi)=1-\left\{1-\left\langle\begin{array}{l}\left(\sum_{t=1}^{h}I_{ijt}^{+L}(\psi)\right)^{\frac{1}{h}}+\left(\sum_{t=1}^{h}I_{jt}^{+L}(\psi)\right)^{\frac{1}{h}}\\ -\left(\sum_{t=1}^{h}I_{ijt}^{+L}(\psi)\right)^{\frac{1}{h}}*\left(\sum_{t=1}^{h}I_{jt}^{+L}(\psi)\right)^{\frac{1}{h}}\end{array}\right\rangle\right\}^{\frac{1}{n}}$$

$$I_i^{+R}(\psi)=1-\left\{1-\left\langle\begin{array}{l}\left(\sum_{t=1}^{h}I_{ijt}^{+R}(\psi)\right)^{\frac{1}{h}}+\left(\sum_{t=1}^{h}I_{jt}^{+R}(\psi)\right)^{\frac{1}{h}}\\ -\left(\sum_{t=1}^{h}I_{ijt}^{+R}(\psi)\right)^{\frac{1}{h}}*\left(\sum_{t=1}^{h}I_{jt}^{+R}(\psi)\right)^{\frac{1}{h}}\end{array}\right\rangle\right\}^{\frac{1}{n}}$$

$$F_i^{+L}(\psi)=1-\left\{1-\left\langle \begin{array}{c}\left(\sum_{t=1}^{h}F_{ijt}^{+L}(\psi)\right)^{\frac{1}{h}}+\left(\sum_{t=1}^{h}F_{jt}^{+L}(\psi)\right)^{\frac{1}{h}}\\ -\left(\sum_{t=1}^{h}F_{ijt}^{+L}(\psi)\right)^{\frac{1}{h}}*\left(\sum_{t=1}^{h}F_{jt}^{+L}(\psi)\right)^{\frac{1}{h}}\end{array}\right\rangle\right\}^{\frac{1}{N}}$$

$$F_i^{+R}(\psi)=1-\left\{1-\left\langle \begin{array}{c}\left(\sum_{t=1}^{h}F_{ijt}^{+R}(\psi)\right)^{\frac{1}{h}}+\left(\sum_{t=1}^{h}F_{jt}^{+R}(\psi)\right)^{\frac{1}{h}}\\ -\left(\sum_{t=1}^{h}F_{ijt}^{+R}(\psi)\right)^{\frac{1}{h}}*\left(\sum_{t=1}^{h}F_{jt}^{+R}(\psi)\right)^{\frac{1}{h}}\end{array}\right\rangle\right\}^{\frac{1}{N}}$$

$$T_i^{-L}(\psi)=-\left\{-\left\langle \begin{array}{c}\left(1-\left(1-\sum_{t=1}^{h}T_{ijt}^{-L}(\breve{\psi})\right)^{\frac{1}{h}}\right)-\left(1-\left(1-\sum_{t=1}^{h}T_{jt}^{-L}(\breve{\psi})\right)^{\frac{1}{h}}\right)\\ -\left(1-\left(1-\sum_{t=1}^{h}T_{ijt}^{-L}(\breve{\psi})\right)^{\frac{1}{h}}\right)*\left(1-\left(1-\sum_{t=1}^{h}T_{jt}^{-L}(\breve{\psi})\right)^{\frac{1}{h}}\right)\end{array}\right\rangle\right\}^{\frac{1}{n}}$$

$$T_i^{-R}(\psi)=-\left\{-\left\langle \begin{array}{c}\left(1-\left(1-\sum_{t=1}^{h}T_{ijt}^{-R}(\breve{\psi})\right)^{\frac{1}{h}}\right)-\left(1-\left(1-\sum_{t=1}^{h}T_{jt}^{-R}(\breve{\psi})\right)^{\frac{1}{h}}\right)\\ -\left(1-\left(1-\sum_{t=1}^{h}T_{ijt}^{-R}(\breve{\psi})\right)^{\frac{1}{h}}\right)*\left(1-\left(1-\sum_{t=1}^{h}T_{jt}^{-R}(\breve{\psi})\right)^{\frac{1}{h}}\right)\end{array}\right\rangle\right\}^{\frac{1}{n}}$$

$$I_i^{-L}(\psi)=-\left\{\left(\sum_{t=1}^{h}I_{ijt}^{-L}(\psi)\right)^{\frac{1}{h}}*\left(\sum_{t=1}^{h}I_{jt}^{-L}(\psi)\right)^{\frac{1}{h}}\right\}^{\frac{1}{n}}$$

$$I_i^{-R}(\psi)=-\left\{\left(\sum_{t=1}^{h}I_{ijt}^{-R}(\psi)\right)^{\frac{1}{h}}*\left(\sum_{t=1}^{h}I_{jt}^{-R}(\psi)\right)^{\frac{1}{h}}\right\}^{\frac{1}{n}}$$

$$F_i^{-L}(\psi)=-\left\{1-\left\langle 1-\left(-\left(\sum_{t=1}^{h}F_{ijt}^{-L}(\psi)\right)^{\frac{1}{h}}*\left(\sum_{t=1}^{h}F_{jt}^{-L}(\psi)\right)^{\frac{1}{h}}\right)\right\rangle^{\frac{1}{n}}\right\}$$

$$F_i^{-R}(\psi)=-\left\{1-\left\langle 1-\left(-\left(\sum_{t=1}^{h}F_{ijt}^{-R}(\psi)\right)^{\frac{1}{h}}*\left(\sum_{t=1}^{h}F_{jt}^{-R}(\psi)\right)^{\frac{1}{h}}\right)\right\rangle^{\frac{1}{n}}\right\}$$

Step 4. Determine $\breve{Y}^{+},\breve{Y}^{-},\breve{d}_i^{+}$ and $\breve{d}_i^{-}$

This section defines a positive-ideal solution (FPIS, $\breve{Y}^{+}$) and a negative ideal solution (FNIS, $\breve{Y}^{-}$):

$$\breve{Y}^{+}=\langle\breve{\psi},\{s_{\max(\partial_{ijt},\chi_{jt})}(\breve{\psi})([1,1],[0,0],[0,0],[-1,-1],[0,0],[0,0])\}\rangle$$

$$\breve{Y}^{-}=\langle\breve{\psi},\{s_{\min(\partial_{ijt},\chi_{jt})}(\breve{\psi})([0,0],[1,1],[1,1],[0,0],[-1,-1],[-1,-1])\}\rangle \quad (12)$$

The distances of $\breve{Y}_i, i=1,\ldots,m$ from $\breve{Y}^{+}$ and $\breve{Y}^{-}$ are defined as:

$$\breve{d}_i^{+}(\breve{Y}_i,\breve{Y}^{+}) = \sqrt{\frac{1}{12(n-1)}\left(\begin{array}{l} \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times T_{Y_i}^{+L}(\breve{\psi}) - \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times T_{Y_i}^{+R}(\breve{\psi}) - \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times I_{Y_i}^{+L}(\breve{\psi})\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times I_{Y_i}^{+R}(\breve{\psi})\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times F_{Y_i}^{+L}(\breve{\psi})\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times F_{Y_i}^{+R}(\breve{\psi})\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times T_{Y_i}^{-L}(\breve{\psi}) + \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times T_{Y_i}^{-R}(\breve{\psi}) + \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times I_{Y_i}^{-L}(\breve{\psi})\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times I_{Y_i}^{-R}(\breve{\psi})\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times F_{Y_i}^{-L}(\breve{\psi})\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times F_{Y_i}^{-R}(\breve{\psi})\right\}^2 \end{array}\right)} \quad (13)$$

$$\breve{d}_i^{-}(\breve{Y}_i,\breve{Y}^{-}) = \sqrt{\frac{1}{12(n-1)}\left(\begin{array}{l} \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times T_{Y_i}^{+L}(\breve{\psi})\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times T_{Y_i}^{+R}(\breve{\psi})\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times I_{Y_i}^{+L}(\breve{\psi}) - \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times I_{Y_i}^{+R}(\breve{\psi}) - \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times F_{Y_i}^{+L}(\breve{\psi}) - \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times F_{Y_i}^{+R}(\breve{\psi}) - \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times T_{Y_i}^{-L}(\breve{\psi})\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times T_{Y_i}^{-R}(\breve{\psi})\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times I_{Y_i}^{-L}(\breve{\psi}) + \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times I_{Y_i}^{-R}(\breve{\psi}) + \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times F_{Y_i}^{-L}(\breve{\psi}) + \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 \\ + \left\{\breve{s}_{\upsilon_i}(\breve{\psi})\times F_{Y_i}^{-R}(\breve{\psi}) + \breve{s}_{\max(\partial_{ijt},\chi_{jt})}\right\}^2 \end{array}\right)} \quad (14)$$

Step 5. Ranking the alternatives.

This study applies a closeness coefficient (CC_i) to rank the alternatives:

$$CC_{Y_i} = \frac{\breve{d}_i^{-}(\breve{Y}_i,\breve{Y}^{-})}{\breve{d}_i^{+}(\breve{Y}_i,\breve{Y}^{+}) + \breve{d}_i^{-}(\breve{Y}_i,\breve{Y}^{-})} \quad (15)$$

The higher value of CC_i, the higher-ranking order of alternatives.

5. Application of the proposed IBL_NS - TOPSIS approach

This section applies the proposed IBL_NS - TOPSIS approach to solve the decision making problem adapted from Sahin and Yigider (2014). In this example, four decision makers $(\breve{D}_1 \square \breve{D}_4)$ have been appointed to evaluate five suppliers $(\breve{Y}_1 \square \breve{Y}_5)$ based on five criteria $(\breve{C}_1 \square \breve{C}_5)$. The computational procedure is summarized as follows:

Step 1. Aggregation of the ratings of suppliers.

Four decision makers determine the suitability ratings of five suppliers versus the criteria using the IBL_NS: *S= {s_1 = Ve_Lo, s_2 = Lo, s_3 = Fa, s_4 = Go, s_5 = Ve_Go}* where VL = Very Low = <(*s_1*, ([0.1, 0.2], [0.6, 0.7], [0.6, 0.7], [-0.8, -0.7], [-0.6, -0.5], [-0.4, -0.3]))>, L = Low = <(*s_2*, ([0.2, 0.3], [0.5, 0.6], [0.6, 0.7], [-0.7, -0.6], [-0.5, -0.4], [-0.4, -0.3]))>, F = Fair = <(*s_3*, ([0.3, 0.5], [0.4, 0.6],[0.4, 0.5], [-0.6, -0.5], [-0.6, -0.4],[-0.6, -0.5]))>, G = Good = <(*s_4*, ([0.5, 0.6], [0.5, 0.6], [0.3, 0.4], [-0.5, -0.4], [-0.5, -0.4], [-0.7, -0.6]))>, and VG = Very Good = <(*s_5*, ([0.6, 0.7], [0.4, 0.5], [0.2, 0.3], [-0.3, -0.2], [-0.6, -0.5], [-0.8, -0.7])).

Board 1 gives the aggregated ratings of five suppliers versus five criteria from four decision makers using Eq. (9) and the data presented in Boards 4-8 in Sahin and Yiğider (2014).

Board 1 Aggregated ratings of suppliers versus the criteria.

Criteria	Suppliers	Decision-makers				Aggregated ratings
		$\breve{D}_1$	$\breve{D}_2$	$\breve{D}_3$	$\breve{D}_4$	
$\breve{C}_1$	$\breve{Y}_1$	Go	Fa	Go	Go	<($s_{3.75}$, ([0.456, 0.577], [0.473, 0.600], [0.322, 0.423], [-0.523, -0.423], [-0.987, -0.966], [-0.997, -0.992]))>
	$\breve{Y}_2$	Go	Go	Fa	Fa	<($s_{3.5}$, ([0.408, 0.553], [0.447, 0.600], [0.346, 0.447], [-0.548, -0.447], [-0.990, -0.966], [-0.996, -0.990]))>
	$\breve{Y}_3$	Lo	Go	Fa	Lo	<($s_{2.75}$, ([0.312, 0.440], [0.473, 0.600], [0.456, 0.560], [-0.619, -0.518], [-0.987, -0.966], [-0.989, -0.975]))>
	$\breve{Y}_4$	Go	Fa	Go	Fa	<($s_{3.5}$, ([0.408, 0.553], [0.447, 0.600], [0.346, 0.447], [-0.548, -0.447], [-0.990, -0.966], [-0.996, -0.990]))>
	$\breve{Y}_5$	Fa	Go	Go	Go	<($s_{3.75}$, ([0.456, 0.577], [0.473, 0.600], [0.322, 0.423], [-0.523, -0.423], [-0.987, -0.966], [-0.997, -0.992]))>
$\breve{C}_2$	$\breve{Y}_1$	Go	Go	Fa	Go	<($s_{3.75}$, ([0.456, 0.577], [0.473, 0.600], [0.322, 0.423], [-0.523, -0.423], [-0.987, -0.966], [-0.997, -0.992]))>
	$\breve{Y}_2$	Go	Fa	Lo	Fa	<(s_3, ([0.335, 0.486], [0.447, 0.600], [0.412, 0.514], [-0.596, -0.495], [-0.990, -0.966], [-0.993, -0.982]))>
	$\breve{Y}_3$	Lo	Go	Go	Go	<($s_{3.5}$, ([0.438, 0.540], [0.500, 0.600], [0.357, 0.460], [-0.544, -0.443], [-0.984, -0.966], [-0.996, -0.989]))>
	$\breve{Y}_4$	Fa	Lo	Go	Lo	<($s_{2.75}$, ([0.312, 0.440], [0.473, 0.600], [0.456, 0.560], [-0.619, -0.518], [-0.987, -0.966], [-0.989, -0.975]))>
	$\breve{Y}_5$	Go	Go	Fa	Go	<($s_{3.75}$, ([0.456, 0.577], [0.473, 0.600], [0.322, 0.423], [-0.523, -0.423], [-0.987, -0.966], [-0.997, -0.992]))>
$\breve{C}_3$	$\breve{Y}_1$	Fa	Fa	Lo	Lo	<($s_{2.5}$, ([0.252, 0.408], [0.447, 0.600], [0.490, 0.592], [-0.648, -0.548], [-0.990, -0.966], [-0.985, -0.968]))>
	$\breve{Y}_2$	Go	Go	Go	Go	<(s_4, ([0.500, 0.600], [0.500, 0.600], [0.300, 0.400], [-0.500, -0.400], [-0.984, -0.966], [-0.998, -0.994]))>
	$\breve{Y}_3$	Lo	Go	Fa	Fa	<(s_3, ([0.335, 0.486], [0.447, 0.600], [0.412, 0.514], [-0.596, -0.495], [-0.990, -0.966], [-0.993, -0.982]))>
	$\breve{Y}_4$	Go	Fa	Go	Fa	<($s_{3.5}$, ([0.408, 0.553], [0.447, 0.600], [0.346, 0.447], [-0.548, -0.447], [-0.990, -0.966], [-0.996, -0.990]))>
	$\breve{Y}_5$	Fa	Go	Go	Go	<($s_{3.75}$, ([0.456, 0.577], [0.473, 0.600], [0.322, 0.423], [-0.523, -0.423], [-0.987, -0.966], [-0.997, -0.992]))>
$\breve{C}_4$	$\breve{Y}_1$	Go	Lo	Fa	Lo	<($s_{2.75}$, ([0.312, 0.440], [0.473, 0.600], [0.456, 0.560], [-0.619, -0.518], [-0.987, -0.966], [-0.989, -0.975]))>
	$\breve{Y}_2$	Go	Go	Lo	Go	<($s_{3.5}$, ([0.438, 0.540], [0.500, 0.600], [0.357, 0.460], [-0.544, -0.443], [-0.984, -0.966], [-0.996, -0.989]))>
	$\breve{Y}_3$	Fa	Fa	Fa	Fa	<(s_3, ([0.300, 0.500], [0.400, 0.600], [0.400, 0.500], [-0.600, -0.500], [-0.994, -0.966], [-0.994, -0.984]))>
	$\breve{Y}_4$	Lo	Lo	Fa	Go	<($s_{2.75}$, ([0.312, 0.440], [0.473, 0.600], [0.456, 0.560], [-0.619, -0.518], [-0.987, -0.966], [-0.989, -0.975]))>
	$\breve{Y}_5$	Fa	Go	Go	Go	<($s_{3.75}$, ([0.456, 0.577], [0.473, 0.600], [0.322, 0.423], [-0.523, -0.423], [-0.987, -0.966], [-0.997, -0.992]))>
$\breve{C}_5$	$\breve{Y}_1$	Lo	Fa	Go	Lo	<($s_{2.75}$, ([0.312, 0.440], [0.473, 0.600], [0.456, 0.560], [-0.619, -0.518], [-0.987, -0.966], [-0.989, -0.975]))>
	$\breve{Y}_2$	Go	Lo	Go	Go	<($s_{3.5}$, ([0.438, 0.540], [0.500, 0.600], [0.357, 0.460], [-0.544, -0.443], [-0.984, -0.966], [-0.996, -0.989]))>
	$\breve{Y}_3$	Go	Go	Lo	Fa	<($s_{3.25}$, ([0.388, 0.514], [0.473, 0.600], [0.383, 0.486], [-0.569, -0.468], [-0.987, -0.966], [-0.995, -0.986]))>
	$\breve{Y}_4$	Lo	Lo	Fa	Go	<($s_{2.75}$, ([0.312, 0.440], [0.473, 0.600], [0.456, 0.560], [-0.619, -0.518], [-0.987, -0.966], [-0.989, -0.975]))>
	$\breve{Y}_5$	Go	Go	Go	Go	<(s_4, ([0.500, 0.600], [0.500, 0.600], [0.300, 0.400], [-0.500, -0.400], [-0.984, -0.966], [-0.998, -0.994]))>

Step 2. Aggregate the importance weights.

The aggregated weights of criteria obtained by Eq. (10) are shown in the last column of Board 2 using the IBL_NS, V = {v_1 = UI, v_2 = OI, v_3 = I, v_4 = VI, v_5 = AI}, where UI = Unimportant = <(v_1, ([0.1, 0.2], [0.4, 0.5], [0.6, 0.7], [-0.8, -0.7], [-0.6, -0.5], [-0.5, -0.4]))>, OI = Ordinary Important = <(v_2, ([0.2, 0.4], [0.5, 0.6], [0.4, 0.5], [-0.7, -0.6], [-0.5, -0.4], [-0.6, -0.5]))>, I = Important = <(v_3, ([0.4, 0.6], [0.4, 0.5], [0.3, 0.4], [-0.6, -0.5], [-0.6, -0.5], [-0.7, -0.6]))>, VI = Very Important = <(v_4, ([0.6, 0.8], [0.5, 0.6], [0.2, 0.3], [-0.5, -0.4], [-0.5, -0.4], [-0.8, -0.7]))>, and AI = Absolutely Important = <(v_5, ([0.7, 0.9], [0.4, 0.5], [0.1, 0.2], [-0.4, -0.3], [-0.6, -0.5], [-0.9, -0.8]))>.

Board 2 The importance and aggregated weights of the criteria.

Criteria	**Decision-makers**				**Aggregated weights**
	$\breve{D}_1$	$\breve{D}_2$	$\breve{D}_3$	$\breve{D}_4$	
$\breve{C}_1$	AI	AI	AI	VI	<($s_{3.75}$, ([0.678, 0.881], [0.423, 0.523], [0.119, 0.221], [-0.423, -0.322], [-0.992, -0.981], [-1.000, -0.999]))>
$\breve{C}_2$	VI	I	I	VI	<($s_{2.5}$, ([0.510, 0.717], [0.447, 0.548], [0.245, 0.346], [-0.548, -0.447], [-0.990, -0.977], [-0.999, -0.996]))>
$\breve{C}_3$	AI	AI	VI	AI	<($s_{3.75}$, ([0.678, 0.881], [0.423, 0.523], [0.119, 0.221], [-0.423, -0.322], [-0.992, -0.981], [-1.000, -0.999]))>
$\breve{C}_4$	VI	VI	I	OI	<($s_{2.25}$, ([0.474, 0.687], [0.473, 0.573], [0.263, 0.366], [-0.569, -0.468], [-0.987, -0.972], [-0.999, -0.995]))>
$\breve{C}_5$	I	I	AI	AI	<(s_3, ([0.576, 0.800], [0.400, 0.500], [0.173, 0.283], [-0.490, -0.387], [-0.994, -0.984], [-1.000, -0.998]))>

Step 3. Aggregate the weighted ratings of suppliers versus criteria.

Board 3 presents the final evaluation values of each supplier using Eq. (11).

Board 3 The final evaluation values of suppliers.

Suppliers	**Aggregated weights**
$\breve{Y}_1$	<($s_{9.038}$, ([0.204, 0.383], [0.698, 0.813], [0.514, 0.649], [-0.400, -0.193], [-1.000, -1.000], [-1.000, -1.000]))>
$\breve{Y}_2$	<($s_{10.8}$, ([0.251, 0.438], [0.704, 0.813], [0.467, 0.609], [-0.347, -0.143], [-1.000, -1.000], [-1.000, -1.000]))>
$\breve{Y}_3$	<($s_{9.363}$, ([0.206, 0.392], [0.693, 0.813], [0.512, 0.648], [-0.404, -0.205], [-1.000, -1.000], [-1.000, -1.000]))>
$\breve{Y}_4$	<($s_{9.513}$, ([0.210, 0.395], [0.695, 0.813], [0.511, 0.648], [-0.411, -0.212], [-1.000, -1.000], [-1.000, -1.000]))>
$\breve{Y}_5$	<($s_{11.588}$, ([0.272, 0.463], [0.704, 0.813], [0.441, 0.585], [-0.310, -0.108], [-1.000, -1.000], [-1.000, -1.000]))>

Step 4. Calculation of $\breve{Y}^+, \breve{Y}^-, \breve{d}_i^+$ and $\breve{d}_i^-$

As shown in Board 4, the distance of each supplier from $\breve{Y}^+$ and $\breve{Y}^-$ can be calculated using Eqs. (12-14).

Board 4 The distance of the suppliers from $\breve{Y}^+$ and $\breve{Y}^-$.

Suppliers	$\breve{d}^+$	$\breve{d}^-$
$\breve{Y}_1$	4.077	1.217
$\breve{Y}_2$	4.468	1.313
$\breve{Y}_3$	4.135	1.216
$\breve{Y}_4$	4.161	1.221
$\breve{Y}_5$	4.666	1.448

Step 5. Obtain the closeness coefficient.

Board 5 presents the closeness coefficients of each supplier using our proposed approach. The ranking order of the five suppliers is $\breve{Y}_1 \succ \breve{Y}_3 \succ \breve{Y}_2 \succ \breve{Y}_4 \succ \breve{Y}_5$. Obviously, the results in Sahin and Yigider (2014) conflict with ours in this paper.

Board 5 Closeness coefficients of suppliers.

Suppliers	Closeness coefficient	Ranking
$\breve{Y}_1$	0.22992	1
$\breve{Y}_2$	0.22707	3
$\breve{Y}_3$	0.22721	2
$\breve{Y}_4$	0.22689	4
$\breve{Y}_5$	0.23687	5

6. Conclusions

Interval bipolar linguistic neutrosophic sets (IBL_NS) are very useful tools in decision making for solving the problem under a vague environment. This paper defined the IBL_NS for decision making under uncertainty situations. Some basic set theoretic operations such as union, intersection, and complement, and the operational rules of IBL_NS were defined. Then, the TOPSIS procedures in IBL_NS were developed. In the proposed TOPSIS approach, aggregate ratings of alternative versus criteria, aggregate the importance weights were expressed in IBL_NS. The closeness coefficient was applied to rank the alternatives. An application was made to demonstrate the advantages of the proposed IBL_NS-TOPSIS approach and compare it with existing methods. The application included detailed calculations which showed that the proposed approach is more general as compared to relevant studies. However, it should be noted that the proposed TOPSIS approach was developed for a static time period. Future research could extend this approach to a dynamic environment. Additionally, the proposed TOPSIS method could also be expanded by using interval bipolar linguistic complex neutrosophic sets.

Ethical considerations

Not applicable.

Conflict of Interest

The author declares that has no conflict of interest.

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