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Reliability approximation analysis of formulated mathematical models in pedal powered bamboo slivering operation

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Abstract This paper presents the reliability approach of formulated mathematical models as well as clubbed models to examine the effects of input factors over the responses of human powered bamboo slicing phenomena. Mathematical models as well as clubbed models were developed for operational time required to cut the bamboo slivers, number or quantity of slivers to be cut and torque (resistive) required for slivering operation run by human powered flywheel motor (HPFM). Experimental data based models evolved represent various responses or output of human powered bamboo slicing phenomena. Design and fabrication of experimental set up was carried out and proper instrumentation was also decided. Design data for sliver cutting was established by the way of performing extensive experimentation. The response data was gathered which was generated by using a wide range of different independent factors by varying them. Performance characteristics were validated on the method of experimentation. The models were optimized. The reliability estimation and analysis of sensitivity was carried out for checking the effect (behavior) of various input parameters with respect to output response parameters. However, this paper entirely presents analysis of reliability evaluation of created models for various operational parameters in human powered bamboo slicing phenomena.

Keywords: human powered flywheel motor, bamboo, frequency distribution, percentage error, response factors, independent factors.

1. Preface of the Work

In present research work the design data is established for sliver cutting from bamboo energized or driven by human powered flywheel motor (HPFM). This design data was used to create the particular slivering unit (unit of sliver cutting). This design data generated through theory of experimentation can be widely and extensively useful for small industrialists, entrepreneurs and semiskilled people etc. Design data is generated and performance validation was carried out based on methods of experimentation approach suggested by Schenck Jr (1968).

Based on the design data, the experimental set-up was fabricated which comprises of basically three units viz. HPFM unit or Energy unit, Power transmission unit comprising gearing and clutch, and Process unit or Bamboo slicing unit as shown in Figures 1 and 2.

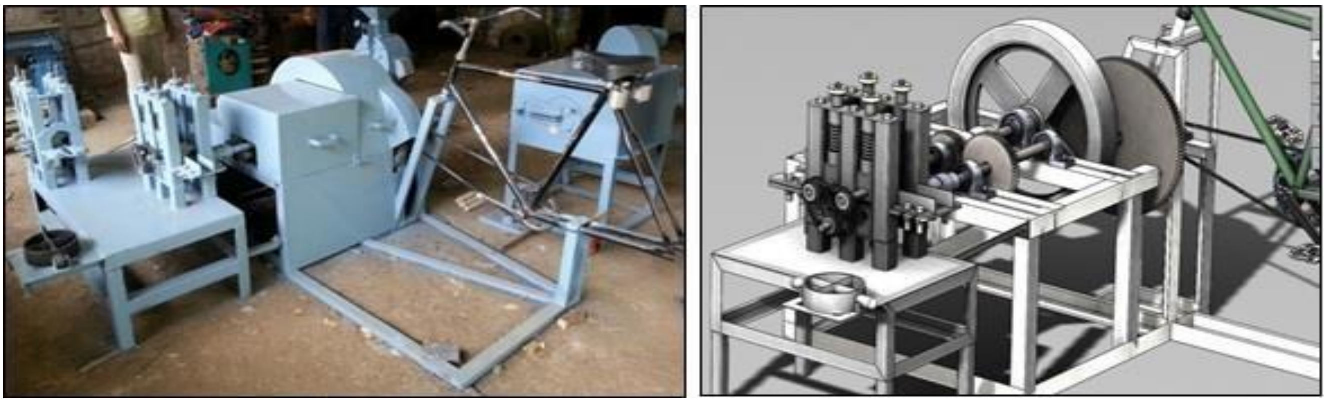

Figure 1 Fabricated Set up & its CAD model.

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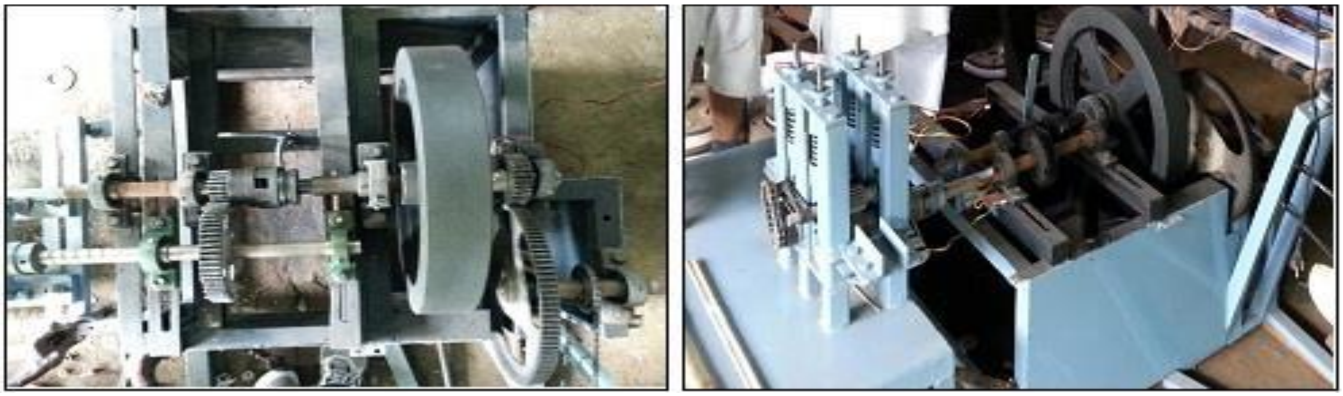

Figure 2 Energy & power transmission unit coupled with process unit.

During the experimental process, the bamboo splits of three different lengths (2.5, 2.0 and 1.5 in ft.) and varying ranges of diameter (50 to 60, 40 to 50 and 30 to 40 in mm) with various widths and thickness were processed in bamboo slivering machine at four distinct speeds (600, 500, 400 and 300 rpm) as well as at three distinct gear ratios of 1/2, 1/3, and 1/4. Hence, numerous bamboo varieties were utilized throughout experiments to analyze the machine's real and functional practicality and its feasibility. Using specially designed electronic kit, the process or operational time, torque (resistive), number or quantity of slivers, flywheel speeding up time etc. were computed (Undirwade 2022).

Methodology of experimentation was applied for design of experiments (Schenck Jr 1968). The input variables were identified like Flywheel Speed (ω_f), flywheel speeding up time (t_f), Flywheel Energy (E_f), Gravity Acceleration (g), Bamboo split thickness (t_b), Bamboo split width (W_b), Bamboo split length (L_b), Roller Center to Cutter Tip distance (L_{rc}), Vertical center distance between roller pairs (C_V), Horizontal center distance between roller pairs (C_H), Cutter Elastic Modulus (E_c), Bamboo Elastic Modulus (E_b), Cutting Angle of Cutter (Φ_c), Gear Ratio (G), and output (or response) factors such as Process or operational time (t_p), Number or quantity of slivers (n), Torque-resistive (T_r) (Undirwade et al 2017). The pi terms were formed for all input and output parameters or factors, reduction of input pi terms was carried out to reduce the complexity of the phenomenon and dimensional equations were formed for all three response variables. Then mathematical models (experimental data based models), clubbed models were formulated for checking their performance and behavior in the phenomenon (Undirwade 2018). Technique of Dimensional Analysis (DA) is used to develop theoretical models and/or predict the behavior of phenomenon, especially when the relation between dependent (response) and independent factors is not clearly known. This theoretical model developed using Dimensional Analysis (DA) can be used to cross check the empirical model developed based on experimental data. Such cross validation helps to ensure the reliability of empirical model developed (Schenck Jr 1968). The formulated mathematical models for response factors such as process or operational time, number or quantity of slivers, torque-resistive (average) and torque-resistive (total) are given in equations 1, 2, 3 & 4 respectively.

$$t_P = 5.15\,X10^{-10}\sqrt{\frac{L_b}{g}}\left\{\left(\frac{E_f}{L_b^3 E_b}\right)^{0.2889}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{0.1564}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{-0.1769}(G)^{-0.3499}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0371}\left(\frac{E_c}{E_b}\right)^{-3.1676}(\varphi_c)^{-30.5644}\right\} \quad (1)$$

$$n = 174.1406\left\{\left(\frac{E_f}{L_b^3 E_b}\right)^{0.5082}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{-0.3148}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{-0.1208}(G)^{-0.5089}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{-0.1823}\left(\frac{E_c}{E_b}\right)^{1.3087}(\varphi_c)^{-1.4871}\right\} \quad (2)$$

$$T_{r-Avg} = 1.08E+10(L_b^3 E_b)\left\{\left(\frac{E_f}{L_b^3 E_b}\right)^{0.7824}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{-1.0005}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{-0.351}(G)^{-1.4423}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0889}\left(\frac{E_c}{E_b}\right)^{4.289}(\varphi_c)^{23.515}\right\} \quad (3)$$

$$T_{r-Total} = 7.68\,X10^{10}(L_b^3 E_b)\left\{\left(\frac{E_f}{L_b^3 E_b}\right)^{0.7188}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{-1.2769}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{0.0101}(G)^{-1.3708}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0265}\left(\frac{E_c}{E_b}\right)^{-7.5718}(\varphi_c)^{6.7265}\right\} \quad (4)$$

Similarly, in clubbed model all the Pi terms i.e. π_1,π_2,$\pi_3\pi_4\pi_5\pi_6$ and $\pi_7$were clubbed i.e. multiplied together and the mathematical clubbed model were subsequently constructed by the application of regression analysis. The similar analysis method was adopted as in the mathematical models formed above for developing the clubbed models for individual pi terms of all response variables. The formulated clubbed models for response factors such as process or operational time, number or quantity of slivers, resistive torque-average and resistive torque-total are given in equations 5, 6, 7 & 8 respectively.

$$t_P\sqrt{\frac{g}{L_b}} = 5654.5754\left\{\left(\frac{E_f}{L_b^3 E_b}\right)\left(\omega_f\sqrt{\frac{L_b}{g}}\right)\left(t_f\sqrt{\frac{g}{L_b}}\right)(G)\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)\left(\frac{E_c}{E_b}\right)(\varphi_c)\right\}^{0.1149} \quad (5)$$

$$n = 56.066\left\{\left(\frac{E_f}{L_b^3 E_b}\right)\left(\omega_f\sqrt{\frac{L_b}{g}}\right)\left(t_f\sqrt{\frac{g}{L_b}}\right)(G)\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)\left(\frac{E_c}{E_b}\right)(\varphi_c)\right\}^{0.1022} \quad (6)$$

$$\left(\frac{T_{rAvg}}{L_b^3 E_b}\right) = 0.00001274\left\{\left(\frac{E_f}{L_b^3 E_b}\right)\left(\omega_f\sqrt{\frac{L_b}{g}}\right)\left(t_f\sqrt{\frac{g}{L_b}}\right)(G)\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)\left(\frac{E_c}{E_b}\right)(\varphi_c)\right\}^{0.2713} \quad (7)$$

$$\left(\frac{T_{rTot}}{L_b^3 E_b}\right) = 0.000014079\left\{\left(\frac{E_f}{L_b^3 E_b}\right)\left(\omega_f\sqrt{\frac{L_b}{g}}\right)\left(t_f\sqrt{\frac{g}{L_b}}\right)(G)\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)\left(\frac{E_c}{E_b}\right)(\varphi_c)\right\}^{0.2718} \quad (8)$$

The models were optimized and validated by means of ANN simulation (Sakhale et al 2010; Rao 1984; Rao 2002) and the values of all response (output) factors were estimated as shown in Table 1.

Table 1 Dependent/Output pi (π) terms calculated by experimental, mathematical and ANN model.

Mean/Error	t_p	n	T_r
Mean experimental	63.75	3.6667	1.10E-08
Mean ANN	57.3356	2.9537	9.43E-09
Mean math. (model)	62.6922	3.6373	1.05E-08
mean_absolute_error_performance_function (MAEPF)	11.5494	0.8717	4.12E-09
mean_squared_error_performance_function (MSEPF)	260.075	1.1088	2.42E-17
Perf	3.46E+03	23.8172	2.25E-16
% Error between model and ANN	8.544285	18.79416	10.26839
% Error between exp. and ANN	10.0618	19.44528	13.95364

The performance of all these models were checked, evaluated and also compared by performing their reliability, R^2-Coefficient of determination and sensitivity analysis. However, this paper presents only the reliability analysis of the models formulated for human powered bamboo slicing phenomenon.

2. Introduction

The term "reliability" refers to the research field that intends to provide numerical values that seeks to quantify the possibility of system failure. The term "reliability", in a very limited sense, is referred as the possibility that a system will successfully complete its goal (Sonde et al 2020). The uniqueness of this research is regarded as crucial since it is applicable to any mathematical model developed. The major goal of reliability analysis is primly to provide an efficient, precise, accurate and effective global approximation while managing the cost of computing as well as forecast accurateness or accuracy (Kernou and Bouafia 2019). One of the requirements for system design phase is to prevent or reduce the likelihood of system failure. The system contains several components, each of which may have multiple failure scenarios. As a result, it is critical to precisely and effectively forecast system dependability during the design phase. System reliability is often treated as the likelihood that the machine or system will carry on its intended objective without problems (Hao and Xiaoping, 2020). With its development, an engineered systems become more interconnected, and function in real-time, thus reliability analysis is crucial to indicate investment and course of action. The analysis of network reliability emphasizes on a question about probability of complex machines with unreliable components and how it will behave or perform with planned specified operational circumstances (Paredes et al 2019).

Many engineering issues encounter hurdles in reliability approximations and system design because of time-varying ambiguities and performing nature. To determine the failures of machine components, Shui et al (2020) mentioned the two different classes wherein one is to determine if the product's extreme value surpasses a key threshold within the targeted lifecycle and second is to determine if the performance surpasses the highest or lowest range of the safe level for the first time throughout the planned lifecycle. Lixiong et al (2018) have stated two applications in engineering with mathematical examples to exemplify the evidence-based reliability analysis technique. It is vital to assess, regulate, and manage uncertainties in order to improve the reliability of engineering machines or structures because uncertainties in engineering analysis are unavoidable, such as environmental fluctuation, variability of material characteristics, and manufacture tolerance. The conventional random reliability technique is based mostly on probability theory, with the accompanying unpredictability of parameters expressed using a probability density function. Xiaoyue and Haiyue (2018), in their study, provided a discrete approximation approach for numerically calculating mission reliability of the systems having time redundancy in execution of mission and in reliability of higher mission. To prevent underestimating mission dependability or reliability, the influence of time redundancy should be included when evaluating the mission reliability of such systems.

Properties of material, loads applied, and structural geometrical parameters of the systems have significant uncertainties. The computation of reliability considers the consequences of these uncertainties and evaluates the structures' failure probability (Bolin and Liyang 2020). Catastrophic outcomes might emerge from industrial system failures. Engineers must cope with the growing uncertainties in order to create safe systems and avoid disastrous outcomes. The chance of failure needs to become exceedingly low for structural safety. Due to their crucial role in safety of the system, reliability approaches and their related applications have generated a significant lot of attention (Kaveh and Eslamlou 2019). Kim and Straub (2019) in their study suggested the two calculation strategies for the effective assessment of sets of reliability analyses related to various time intervals. In the general scenario, calculating a first-passage probability is necessary for the precise lifetime reliability analysis of decaying structures which might have high expenses of computation. The difficulty can be approximated by breaking it down into a sequence of time-invariant reliability issues with discrete intervals of time. Improving the quality of a multidimensional system necessitates a series of sophisticated design changes. A growing system's complexity may lead to a rise in frequency breakdown. The random and simultaneous incidence of failures or defects in a machine or system might be the primary cause of equipment performance decline. One of the strategies used to predict the lifespan of a machine and its components with many failure factors is theoretical probability distribution. A Weibull distribution which is exceptionally adaptable represents one of the most widely used statistical methods to estimate reliability (Bala et al 2018).

For time-limited tests of higher reliability and longer life systems & components, relatively minimal data of failure may be gathered. Some current approaches fail to achieve confidence interval of reliability factors and point estimation with very less data of failure. The findings are not reliable if confidence interval of reliability factors and point estimation are determined using different approaches. Zhang et al (2019) in their work developed Bayesian reliability evaluation approach for relatively lesser failure data by the use of Weibull distribution. The paper of Sabet et al (2020) highlights upon RelyFSM, a framework for state-level reliability assessment in Finite State Machines (FSM) computations. Understanding the FSM computation reliability in unstable contexts is critical for their essential functions in computing. The increasing trend of soft error rates in newest computer architecture is magnified in approximate computing where unstable hardware components are incorporated to enhance efficiency by simplifying the design of system. Radermacher and Unger (2020) proposed PGD reliability analysis wherein the structural computation solution is produced simply by assessing the PGD solution for specified set of variables without running complete finite element simulation. They presented an efficient structural reliability analysis that took advantage of the benefits of model reduction approaches to decrease the computing effort of assessing the limiting state function. This proposed method provides the path for the use of fully probabilistic methodologies in industrial applications. The environmental considerations, properties of material, external loads and dimension of various components are design parameters. They can be categorized using statistical methods. The deterministic method finds and establishes a worst-case scenario or extreme value to fulfil the design. A probabilistic method employs statistics categorization to offer the needed reliability in the design. The importance of reliability concerns extends beyond static analysis to stability analysis and dynamic analysis as well. One of the newest trends is the utilization of NN (neural networks) in hybrid reliability analysis techniques, which falls under the category of soft computational techniques (Dudzik and Beata 2019). The study by Lin and Shao (2021) established a model for the reliability evaluation as well as residual life evaluation of gas pipelines having various corrosion pits. They presented the reliability approach of Hamiltonian Monte Carlo subset simulation having benefits of high accuracy, minimum cost and lesser sampling. The reliability evaluation and estimated remained service duration of such pipeline are crucial once it has been in use for particular length of time. By treating the input characteristics as random variables and analyzing their impact on the output response or result, the reliability technique should be employed to account for fluctuations. The random input or dependent parameters and output responses or results must have a mathematical connection in order to follow the reliability approach. Reliability connections are clear in the analytical study and are provided by traditional equations for the settlement and sustaining capacity constraints or issues (Lafifi and Rouaiguia 2021).

Wang et al (2022) studied in their work, the issues of correlation and complexity during Approximate Computing Circuit (ACC) reliability evaluation. They provide iterative Probabilistic Transfer Matrix-based ACC reliability evaluation techniques that are quite accurate and comprehensive. With the rising significance and availability of approximate computing circuit (ACC) reliability evaluation, there is high necessity of reliability assessment methods to assess their fault vulnerability. Two precise reliability assessment techniques for approximation computing circuits have been provided for computation of reliability. The iterative Probabilistic Transfer Matrix model (PTM model) is used to determine the dependability of approximation computing circuits. Many variables in a system that may be viewed as random variables and that follow a particular statistical distribution include applied loads, resistance characteristics, material attributes, and dimensional factors. For estimating failure probability, the First Order Reliability Methodology offers the most ease, especially for the relatively small or compact structure. Using a more challenging calculation method, the Second Order Reliability Methodology could offer a result that is comparatively reliable and accurate (Wang 2022). Uncertainties in constructional dimensions, interference due to change in environment, properties of material, etc. are common and unavoidable, and they are major source of instability and even system performance failure. The industry's growing need for lighter, economical and efficient systems makes it increasingly crucial demanding to analyze system reliability while taking uncertainties into account. Many reliability computation techniques and mathematical statistics based probabilistic methods have been employed to address the system's constructional uncertainties. The probability techniques analyze the dependability and system safety through the computation of the probability statistical theory and utilize the precise probability distribution function (PDF) to express the unpredictability of the factors. It is essential to examine effective system reliability analysis techniques employing non-probabilistic approaches. This analysis is helpful to acquire more satisfactory reliability analysis results for the critical structural reliability (Li and Liu 2022). The goal or objective of reliability assessment is to find out the likelihood such that a system shall survive in an unknown situation or fail in that situation. One of the most common uncertain models in the reliability evaluation field is the model of probability, in which the structural uncertainties are characterized as random parameters. The reliability techniques simulate the functional performance at MPP (i.e., most probable point) of limit-state surface which has the higher most probability density. The structural system reliability evaluation has been given a lot of thought over the past years due to the emphasis on system security, and it is now unquestionably crucial in the system design process. The likelihood of failure sensitivity values in relation to the probability distribution are accepted for the random variables (Zakaria et al 2017).

Thus, keeping view towards the significance of analysis of reliability estimation, the present paper specifically highlights upon reliability approximations of formulated models for bamboo slivering process to evaluate the behavior of various operational parameters involved in the human powered bamboo slicing phenomena. The signified importance of this research is regarded as crucial since it is applicable to any mathematical model developed. The error frequency distribution for created models was carried out using a graphical depiction and comparison of these graphs was carried out with commonly and frequently used probability density function graphs of life distributions.

3. Model Reliability Approximation

Plotting graphs of the error frequency distribution for created models is method for assessing a model's reliability. The probability density functional graphs of commonly and frequently used life distributions were compared to these graphs. Frequency distribution is most common and general case in statistical computing or analysis. Different reliability factors and characteristics are modelled using a variety of statistical distributions (Irwin and Marylees 2003). The specific distribution that is employed depends upon the type of data being examined and evaluated. By the comparison of error frequency graphical representation of different mathematical models with probability density functional graphs of commonly and frequently applied life distribution, the reliability approximation of model was executed (Ebeling 2004).

3.1. Frequency distribution of Error for mathematical model

In this experiment, there were observed and computed sets of values for the dependent factors or parameters viz. process or operational time, number or quantity of slivers and torque (resistive) of manual bamboo sliver cutting operation. The difference of observed set of value and computed set of value is an error. Frequency rates of incidence or occurrence for particular errors were evaluated for three models of response factors in bamboo sliver cutting operation as representative sample. Tables 2, 3, 4 and 5 show the frequency distribution of error for response factors' mathematical models of π_{D1}, π_{D2}, $\pi_{D3(Avg.)}$ and $\pi_{D3(Total)}$ respectively.

Table 2 Frequency distribution of error for response factor, process or operational time (mathematical model of π_{D1}).

% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)	% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)
0	2	0	20	1	20
1	5	5	21	4	84
2	4	8	22	1	22
3	5	15	23	1	23
4	4	16	25	1	25
5	4	20	26	2	52
6	4	24	28	1	28
7	5	35	29	1	29
8	6	48	31	3	93
9	2	18	32	1	32
10	4	40	33	2	66
11	4	44	34	1	34
12	7	84	35	1	35
13	2	26	39	1	39
14	5	70	42	1	42
15	8	120	44	1	44
16	3	48	50	1	50
17	5	85	61	1	61
18	1	18	74	1	74
19	2	38	**859**	**108**	**1615**

Error (Mean)	$\sum \frac{fixi}{xt}$	14.953704
Reliability	$\mathbf{100 - Error\ (Mean)}$	85.046296 %

Table 3 Frequency distribution of error for response factor, number or quantity of slivers (mathematical model of π_{D2}).

% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)	% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)
0	8	0	16	5	80
1	7	7	17	4	68
2	7	14	18	3	54
3	5	15	19	2	38
4	5	20	20	1	20
5	3	15	21	2	42
6	5	30	23	2	46
7	2	14	24	2	48
8	6	48	26	1	26
9	2	18	28	1	28
10	6	60	29	1	29
11	6	66	32	1	32
12	6	72	34	1	34
13	5	65	36	1	36
14	5	70	56	1	56
15	2	30	**519**	**108**	**1181**

Error (Mean)	$\sum \frac{fixi}{xi}$	10.9351852
Reliability	$\mathbf{100 - Error\ (Mean)}$	89.0648148 %

Table 4 Frequency distribution of error for response factor, resistive torque- average (mathematical model of $\pi_{D3(Avg.)}$).

% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)	% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)
0	1	0	10	8	80
1	3	3	11	6	66
2	9	18	12	7	84
3	2	6	13	1	13
4	5	20	14	1	14
5	13	65	15	2	30
6	22	132	16	2	32
7	14	98	24	1	24

8	13	104			
9	18	162	**160**	**128**	**951**

Error (Mean)	$\sum \frac{fixi}{xi}$	7.429688
Reliability	$100 - \text{Error (Mean)}$	92.57031 %

Table 5 Frequency distribution of error for response factor, resistive torque- total (mathematical model of $\pi_{D3(Total)}$).

% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)	%Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)
0	65	0	46	3	138
1	71	71	47	1	47
2	54	108	48	2	96
3	61	183	49	2	98
4	62	248	50	1	50
5	70	350	51	1	51
6	58	348	52	1	52
7	58	406	53	2	106
8	68	544	54	2	108
9	53	477	55	2	110
10	45	450	57	2	114
11	40	440	58	2	116
12	51	612	60	1	60
13	53	689	61	1	61
14	47	658	62	2	124
15	46	690	63	2	126
16	35	560	64	1	64
17	36	612	65	4	260
18	34	612	66	1	66
19	33	627	67	3	201
20	30	600	68	3	204
21	22	462	69	1	69
22	23	506	70	4	280
23	33	759	71	4	284
24	11	264	72	3	216
25	20	500	73	4	292
26	11	286	74	1	74
27	9	243	75	2	150
28	8	224	76	1	76
29	9	261	77	3	231
30	11	330	78	1	78
31	16	496	79	2	158
32	7	224	80	1	80
33	3	99	81	1	81
34	5	170	83	1	83
35	8	280	84	2	168
36	4	144	85	1	85
37	2	74	86	4	344
38	2	76	87	1	87
39	4	156	89	1	89
40	3	120	91	1	91
41	3	123	95	1	95
42	4	168	98	1	98
43	3	129	99	1	99
44	3	132	100	3	300
45	2	90	**4203**	**1380**	**21461**

Error (Mean)	$\sum \frac{fixi}{xi}$	15.551449
Reliability	$100 - \text{Error (Mean)}$	84.448551

3.2. Error frequency distribution for clubbed model

Tables 6, 7, 8 and 9 show the frequency distribution of error for response factors' clubbed models of π_{D1}, π_{D2}, $\pi_{D3(Avg.)}$ and $\pi_{D3(Total)}$ respectively.

Table 6 Frequency distribution of error for response factor, process or operational time (clubbed model of π_{D1}).

% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)	% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)
0	2	0	24	3	72
1	6	6	25	2	50
2	1	2	26	2	52
3	1	3	27	3	81
4	2	8	28	4	112
5	1	5	29	2	58
6	3	18	30	1	30
7	4	28	31	1	31
8	2	16	33	1	33
9	6	54	35	2	70
10	4	40	36	4	144
11	4	44	37	1	37
12	4	48	39	1	39
13	2	26	43	1	43
14	9	126	44	1	44
15	2	30	45	1	45
16	4	64	47	2	94
18	3	54	48	2	96
20	4	80	56	2	112
21	2	42	59	2	118
22	3	66			
23	1	23	**982**	**108**	**2144**

Error (Mean)	$\sum \frac{fixi}{xi}$	19.8518519
Reliability	$100 - \text{Error (Mean)}$	80.1481481

Table 7 Frequency distribution of error for response factor, number or quantity of slivers (clubbed model of π_{D2}).

% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)	% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)
0	8	0	24	1	24
1	5	5	26	1	26
2	3	6	27	1	27
3	3	9	28	1	28
4	6	24	29	1	29
5	4	20	30	1	30
6	4	24	33	2	66
7	3	21	34	1	34
8	5	40	35	2	70
9	2	18	36	1	36
10	6	60	41	1	41
11	5	55	42	2	84
12	4	48	43	1	43
13	5	65	47	2	94
14	4	56	53	1	53
15	2	30	54	1	54
16	1	16	55	1	55
17	2	34	63	1	63
18	2	36	68	1	68
19	4	76	75	1	75
20	2	40	100	1	100
22	1	22			
23	2	46	**1198**	**108**	**1851**

Error (Mean)	$\sum \frac{fixi}{xi}$	17.1388889
Reliability	$100 - \text{Error (Mean)}$	82.8611111

Table 8 Frequency distribution of error for response factor, average torque (resistive) (clubbed model of $\pi_{D3(Avg.)}$).

% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)	% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)
0	6	0	19	2	38
1	4	4	20	2	40
2	10	20	21	3	63
3	8	24	22	2	44
4	7	28	23	1	23
5	6	30	25	3	75
6	9	54	26	3	78
7	6	42	28	2	56
8	2	16	30	1	30
9	6	54	33	3	99
10	2	20	34	1	34
11	3	33	35	2	70
12	3	36	36	1	36
13	6	78	41	1	41
14	5	70	43	1	43
15	2	30	46	1	46
16	3	48	48	1	48
17	7	119	59	1	59
18	2	36	**760**	**128**	**1665**

Error (Mean)	$\sum \frac{fixi}{xi}$	13.00781
Reliability	$100 - \text{Error (Mean)}$	86.99219

Table 9 Frequency distribution of error for response factor, total torque (resistive) (clubbed model of $\pi_{D3(Total)}$).

% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)	% of Error f_i (1)	Frequency x_i (2)	$f_i x_i$ (3)
0	26	0	50	22	1100
1	14	14	51	18	918
2	19	38	52	21	1092
3	11	33	53	17	901
4	14	56	54	17	918
5	18	90	55	21	1155
6	18	108	56	17	952
7	13	91	57	16	912
8	17	136	58	11	638
9	13	117	59	13	767
10	15	150	60	10	600
11	11	121	61	16	976
12	21	252	62	9	558
13	21	273	63	8	504
14	13	182	64	4	256
15	11	165	65	12	780
16	22	352	66	5	330
17	16	272	67	11	737
18	15	270	68	5	340
19	17	323	69	9	621
20	9	180	70	4	280
21	12	252	71	4	284
22	15	330	72	11	792
23	20	460	73	6	438
24	8	192	74	7	518
25	22	550	75	3	225
26	19	494	76	1	76
27	14	378	77	9	693
28	18	504	78	5	390
29	19	551	79	4	316
30	19	570	80	6	480
31	11	341	81	3	243
32	19	608	83	5	415

33	15	495	84	4	336
34	18	612	85	4	340
35	17	595	86	4	344
36	21	756	87	2	174
37	21	777	89	5	445
38	16	608	90	1	90
39	11	429	91	6	546
40	16	640	92	4	368
41	16	656	93	1	93
42	18	756	94	2	188
43	10	430	95	4	380
44	18	792	96	2	192
45	7	315	97	3	291
46	21	966	98	2	196
47	10	470	99	4	396
48	16	768	100	204	20400
49	17	833	**4880**	**1380**	**64335**

Error (Mean)	$\sum \frac{\boldsymbol{fixi}}{\boldsymbol{xi}}$	46.61957
Reliability	$\mathbf{100 - Error\ (Mean)}$	53.38043

4. Establishing the Model's reliability

4.1. Establishing the Mathematical Model's reliability

The mathematical formula for reliability is:

$$R = 1 - MeanError$$

and computation of error (mean) is calculated by the formula:

$$MeanError = \frac{\sum f_i \times x_i}{\sum f_i}$$

Tables 2, 3, 4 and 5 show the reliability estimation or calculations in bamboo sliver cutting operation by HPFM for mathematical models of entire three response or dependent π-terms.

Here, f_i - Frequency (occurrence frequency) of occurrence, and
x_i- Percentage of error.

The total system reliability for the system in series is depicted by the equation:

Reliability of System,

$$R_s = 1 - \prod_{i=1}^{n} (R_i)$$

Computation of reliability of system in parallel is indicated by the equation:

$$R_p = 1 - \prod_{i=1}^{n} (1 - R_i)$$

Hence, for such case total system reliability in parallel is provided by:

$$R_p = 1 - \prod_{i=1}^{n} (1 - R_i) = 1 - \{(1 - R_{itp})(1 - R_{in})(1 - R_{iTr})\}$$

Where, R_p - total system reliability in parallel
R_{itp} - reliability for model of process or operational time (π_{D1}),
R_{in} - reliability for model of number or quantity of slivers (π_{D2}), and
R_{iTr} - reliability for model of torque-resistive (π_{D3})

Since during the experimental process, simultaneous observations were recorded and identified;
Thus, for the models of bamboo slivering phenomena, the total reliability is:

$= 1 - \{(1 - 0.850462)(1 - 0.890648)(1 - 0.9257031)\} = 0.9987817 = 99.8781$ %

(Here the value of average torque (resistive) is substituted as 0.9257031 against R_{iTr})

Therefore, Reliability of machine or system = $\mathbf{99.8781}$ %

If the value of total torque (resistive) is substituted as $R_{iTr} =$ 84.448551 %,
Then, for the models of bamboo slivering phenomena, the total reliability is:

$= 1 - \{(1 - 0.850462)(1 - 0.890648)(1 - 0.844485)\} = 0.997456 = 99.7456$ %

Therefore, Reliability of machine or system = $\mathbf{99.7456}$ %

4.2. Establishing the Clubbed Model's reliability

The clubbed model's reliability for the machine is established in the same fashion like mathematical models. Tables 6, 7, 8 and 9 show the reliability estimation or calculations in bamboo sliver cutting operation by HPFM for clubbed models of entire three response or dependent π-terms.

Thus, for the models of bamboo slivering phenomena, the total reliability is:

$= 1 - \{(1 - 0.801481)(1 - 0.828611)(1 - 0.8699219)\} = 0.995574 = 99.5574$ %

(Here the value of average torque (resistive) is substituted as 0.8699219 against R_{iTr})

Therefore, Reliability of machine or system = **99.5574** %

If the value of total torque (resistive) is substituted as R_{iTr} = 53.38043 %,

Then, for the models of bamboo slivering phenomena, the total reliability is:

$= 1 - \{(1 - 0.801481)(1 - 0.828611)(1 - 0.5338043)\} = 0.984138 = 98.4138$ %

Therefore, Reliability of machine or system = **98.4138** %

5. Reliability approximation of model

5.1. Reliability approximation of mathematical model

The reliability of the developed models can be approximated similar to the reliability of the standard life distribution. On the basis of frequency distribution analysis of sample models for three approaches, graphs were plotted for frequency Vs error. Figures 3, 4, 5 and 6 depict the graph of frequency Vs error for various mathematical models of the human powered bamboo sliver cutting operation.

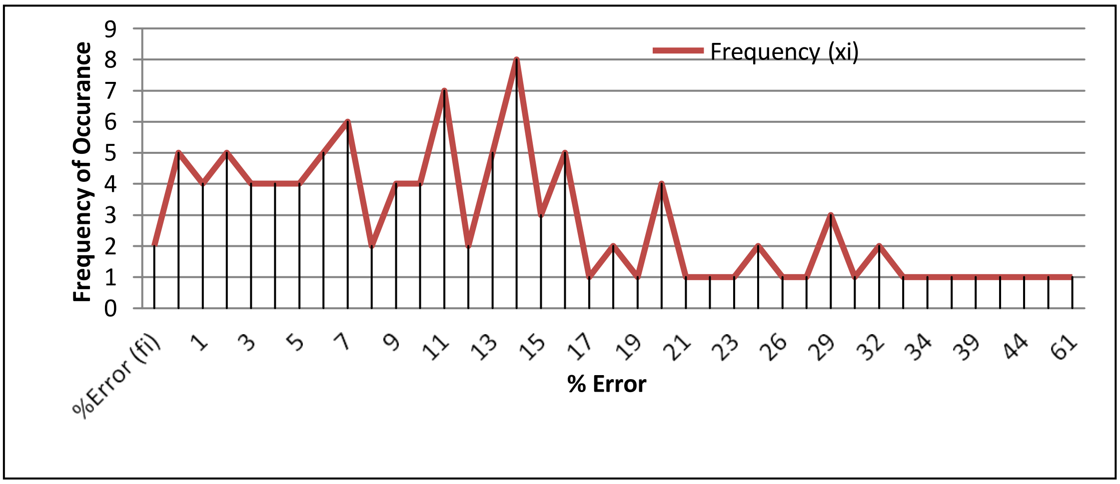

Figure 3 Error frequency graph of mathematical model for processing time, tp (π_{D1}).

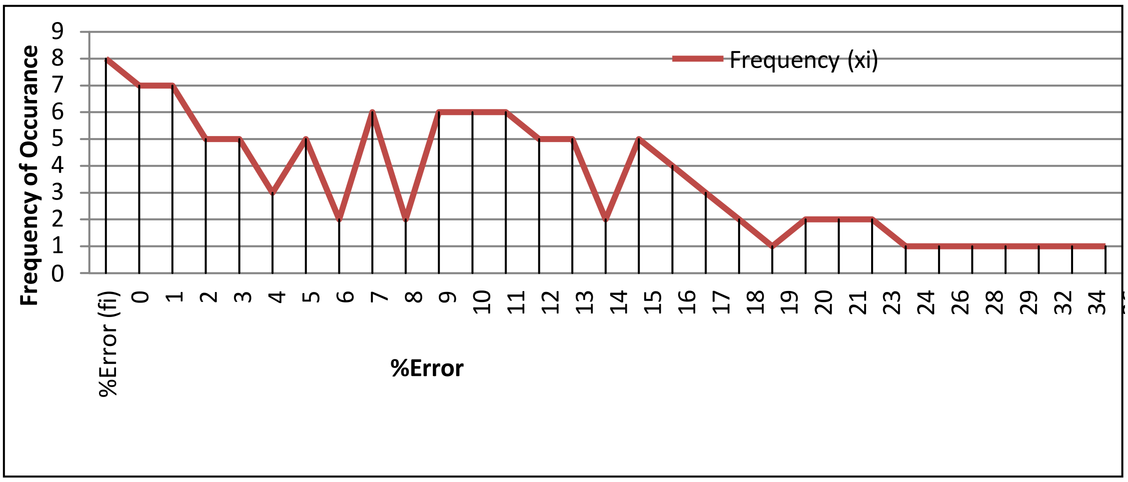

Figure 4 Error frequency graph of mathematical model for number or quantity of slivers, n (π_{D2}).

The comparison of these error frequency graphs was carried out with graphs of probability density function of standard distributions. It is noticed that different portion of the graph confirmed to some of standard distribution i.e. normal, lognormal, exponential and Weibull. These distributions have different reliability. Thus reliabilities of these models as well as these distributions are equal.

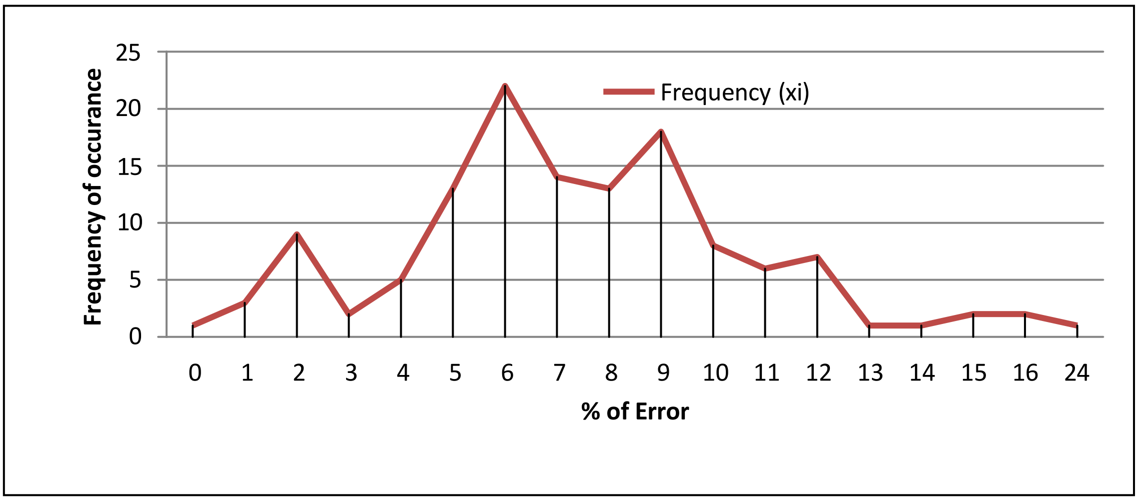

Figure 5 Error frequency graph of mathematical model for resistive torque-average, $T_{r\text{-}avg}$ ($\pi_{D3\text{-}Avg}$).

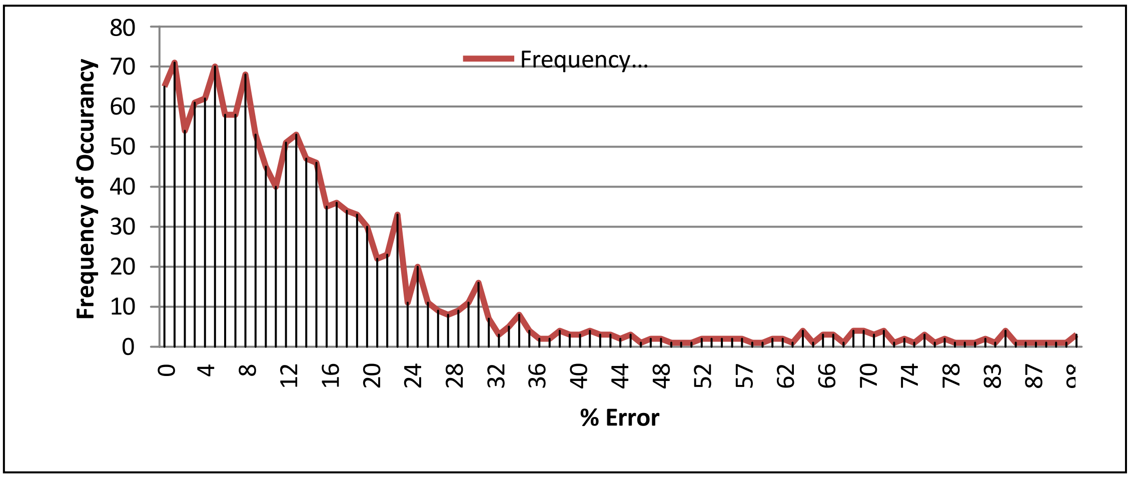

Figure 6 Error frequency graph of mathematical model for resistive torque-total, $T_{r\text{-}total}$ ($\pi_{D3\text{-}Total}$).

5.2. Reliability approximation of clubbed model

Figures 7, 8, 9 and 10 depict the graph of frequency Vs error for various clubbed models of the bamboo sliver cutting operation by HPFM.

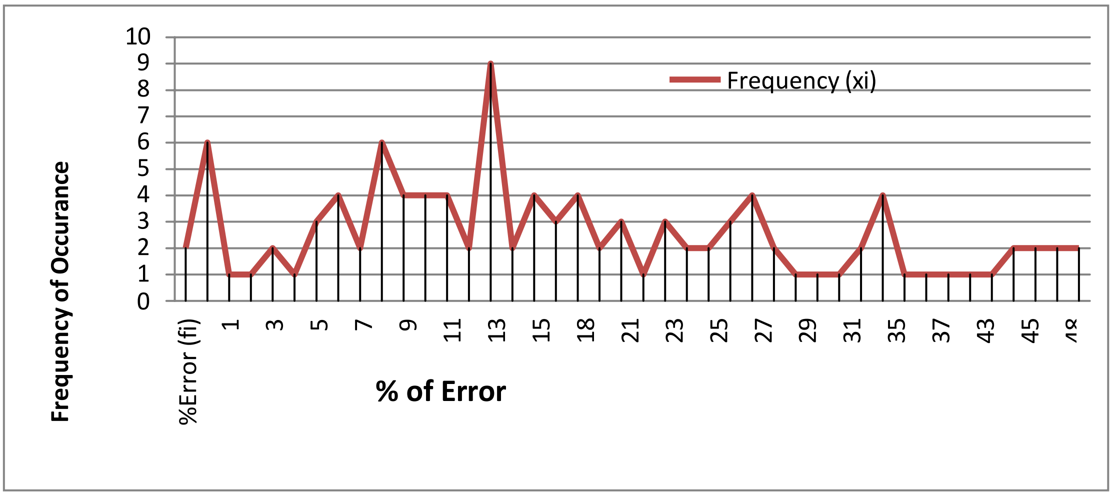

Figure 7 Error frequency graph of clubbed model for processing time, t_p (π_{D1}).

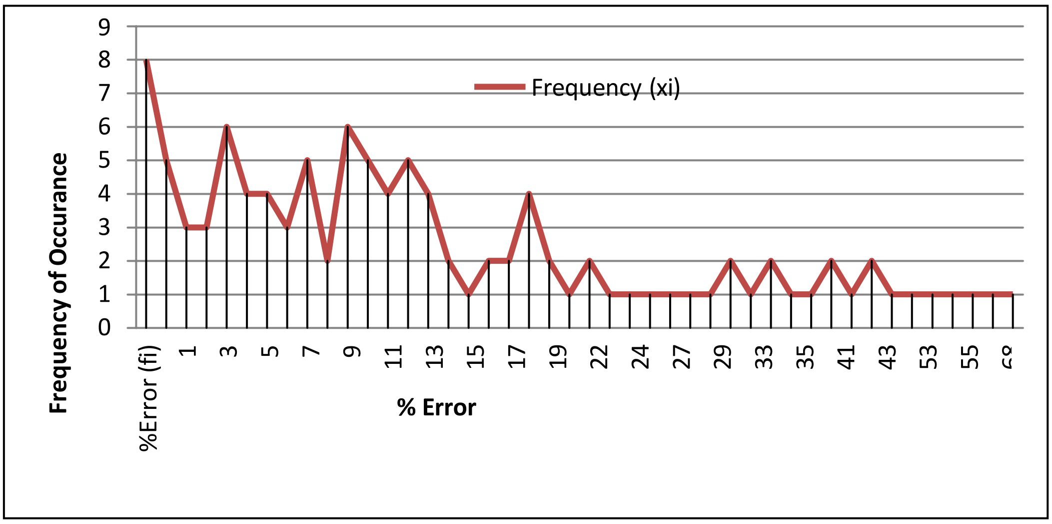

Figure 8 Error frequency graph of clubbed model for number of slivers, n (π_{D2}).

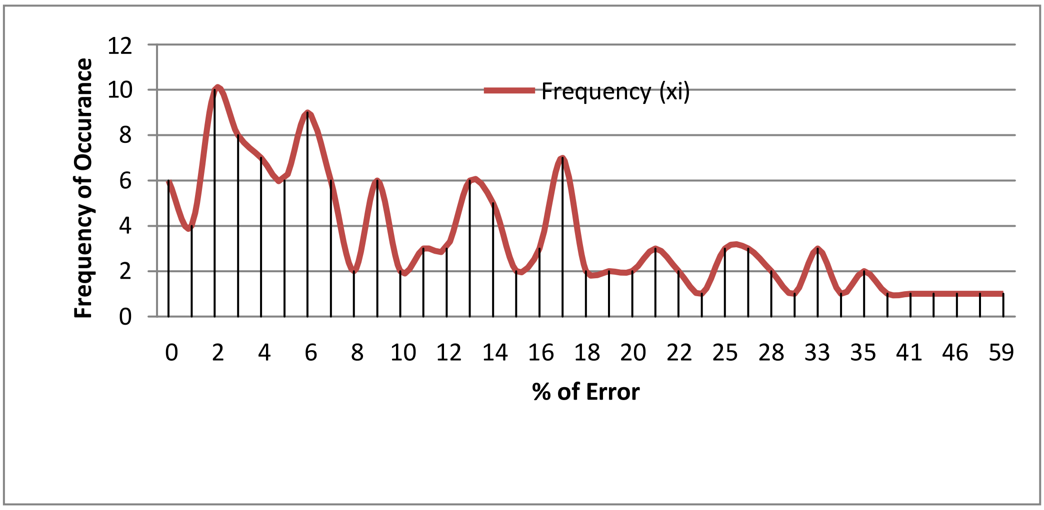

Figure 9 Error frequency graph of clubbed model for resistive torque-average, $T_{r\text{-}avg}$ ($\pi_{D3\text{-}Avg}$).

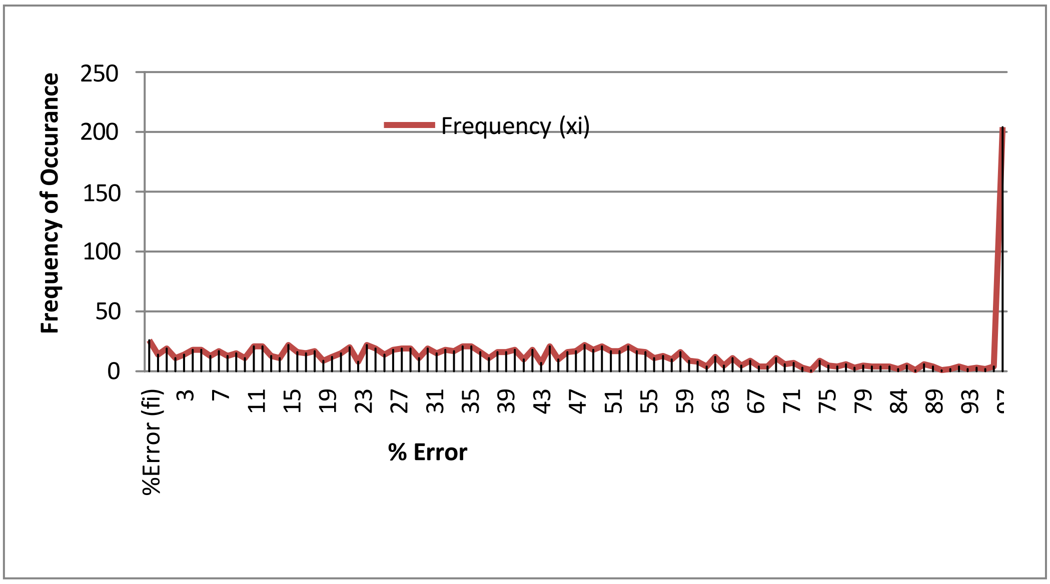

Figure 10 Error frequency graph of clubbed model for resistive torque-total, $T_{r\text{-}total}$($\pi_{D3\text{-}Total}$).

The mathematical models' reliability and clubbed models' reliability are compared and is given in following Table 10.

Table 10 Comparison of the % Reliability for mathematical models and clubbed models of π_{D1}, π_{D2}, and π_{D3}.

π term	% Reliability for mathematical models	% Reliability for clubbed models
π_{D1} (Operational Time)	85.0462	80.1481
π_{D2} (Number or quantity of slivers)	89.0648	82.8611
π_{D3-Avg} i.e. average resistive torque	92.57031	86.99219
$\pi_{D3-Total}$ i.e. total resistive torque	84.4485	53.38043
System Reliability when T_{r-avg} is taken into consideration	99.8781	99.5574
System Reliability when $T_{r-total}$ is taken into consideration	99.7456	98.4138

6. Conclusions

This paper has emphasized on the reliability evaluation approach of mathematical models and clubbed models formulated for the manually driven bamboo slicing machine. Reliability approximation is based on the standard life distribution and associated probability density functions. From the comparative analysis of graphs of error frequency with probability density function it is observed that the models' reliability and the reliability of standard distributions are equivalent. The comparison of percentage reliability of mathematical as well as clubbed models is made. From this comparative data, it is revealed that reliability concerns of mathematical models are found better than that of approach of clubbed models.

From table 10, it is noted that the reliability of mathematical model for processing time (π_{D1}) is 85.0462 %, whereas for clubbed model is 80.1481 %, the reliability of mathematical model for number of slivers (π_{D2}) is 89.0648 %, whereas for clubbed model is 82.8611 %, the reliability of mathematical model for average resistive torque (π_{D3-Avg}) is 92.57031 %, whereas for clubbed model is 86.99219 % and the reliability of mathematical model for total resistive torque ($\pi_{D3-Total}$) is 84.4485 %, whereas for clubbed model is 53.38043 %. Here, our mathematical models formulated are better reliable than clubbed models. The total machine or system reliability is found to be 99.8781 % and 99.5574 % for mathematical models and clubbed models respectively in case of resistive torque-average (T_{r-avg}) whereas it is found to be 99.7456 % and 98.4138 % for mathematical and clubbed models respectively in case of total resistive torque ($T_{r-total}$).

There is very little standardized error in the estimation of predicted and/ or computed values of response or dependent factors. This adds credibility to the mathematical models that have been established. It has been seen from the percentage error values that the mathematical models may be employed effectively for computing of response factors with respect to given collective set of distinct independent factors. The reliability analysis of formulated models for this manually driven bamboo slicing machine proved to be robust in construction to process any sizes and/or varieties of bamboo. The study of this analysis reveals that the economic or financial concerns, feasibility and designing data of this machine is authentic and useful for entrepreneurs, however it may vary from place to place depending on the working condition and environment since this model formulation was carried out considering Indian environmental conditions and species of bamboo. The experimental data in this study was gathered through carrying out actual genuine experiments, therefore the findings of this study accurately reflect the interaction level between different independent factors. The behavior pattern of the mathematical models and clubbed models demonstrated for graphical reliability analysis proves the authenticity of the formulated mathematical models for human powered bamboo slivering operation. The scope may be highlighted such as the obtained models of this study were formulated considering local conditions of manufacturing facilities, local available bamboo samples, workmanship of fabrication and environmental conditions. The adopted models can be applied to other working and environmental conditions and the behavior of these models can be confirmed.

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Ethical considerations

Not applicable.

Conflict of Interest

The author declares no conflicts of interest.

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