Slope stability analysis using probabilistic approach

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Abstract Monte Carlo simulation is a method for calculating a deterministic model iteratively using sets of random numbers as inputs, it's was used widely in probabilistic analysis of slope stability determining the minimum safety factor and predicting of slope instability. This paper presents a method of slope stability analysis based on the Hovland method and using the Monte Carlo simulation that defined two parameters as random variable, the cohesion and the friction angle of the soil. In order to evaluate the efficiency and applicability of the proposed method, an example of a slope located in northern Morocco is presented in this study. The results show that the minimum safety factors calculated in this study are very close to those obtained by other researchers, which also revealed that the present method could be applied to predict the stability of homogenous slopes.

Keywords: Monte Carlo, simulation, safety factor, probabilistic, random variable

1. Introduction

In addition to the recognition of the site and the choice of the mechanical characteristics of the soil, the study of a slope includes a stability calculation to determine on the one hand the failure curve along which the landslide risk is highest and on the other hand the corresponding value of the safety factor. The estimation of slope stability using limit equilibrium methods with respect to risk of failure is one of the important problems in geotechnics especially in the field of limited or unknown data.

In fact, several approaches were used to analyze the slope stability that are deterministic and probabilistic. The deterministic approach is based on the use of fixed values assigned to the parameters that appear in the equations of the mathematical model adopted for the calculation of the safety factor. The variability and uncertainty of the data are not considered in the analysis, thus producing unique results. Contrary to the deterministic approach, the probabilistic approach allows to take into account the variability of all input parameters (such as the mechanical characteristics of the soil) simultaneously in the stability calculation. To do this, a statistical distribution must be assigned to the calculation parameters considered as random variables to which a location in space is associated.

Since the 1980s, optimization methods have been of great interest to geotechnical researchers in solving complex problems. They have shown their effectiveness for slope stability calculation and especially in identification of the critical slip surface. Sambridge et al (2002) indicated that for the complex problems the Monte Carlo methods are more favorable than deterministic methods. The Monte Carlo simulation is easy to implement and can get the minimum safety factor with the desired precision, while it takes a long time due to the high computational effort (Fang et al 2020). However, there are several studies demonstrating the practicality and the efficient of the probabilistic method on the slope stability analysis (Aizhao et al 2022; Joni et al 2022; Nurul 2019).

McCombie et al (2002) developed a simple genetic algorithm for finding the minimum safety factor for circular failure surface in slope stability analysis. They present three parameters: the coordinates of the circle center and the radius of the circular failure surface. They demonstrated that replacing the radius by the coordinates of a point belonging to the circle of rupture, works well, because the formulation of the problem becomes close to what the objective function determines for each individual.

Selmi et al (2007) have developed a calculation code based on the Monte Carlo simulation, allows the slope stability analysis and taking into account the spatial variability of the input parameters, it provides as a result: the critical circle, the probability of failure and the reliability index.

Halder et al (2020) investigated the stability of a reinforced 2D slope, considering the spatial variability and randomness of soil strength parameters. Log-normally distributed isotropic and anisotropic random fields were sampled using the Monte Carlo simulation method. Wang et al (2021) developed a convolutional neural network based network for predicting the factor of safety of a 2D slope with Monte Carlo simulation generated random field realizations of material properties.

This paper presents a calculation model for slope stability analysis using Monte Carlo simulation and based on the Hovland method. The random variables considered in this study are the cohesion and the friction angle of the soil. The proposed model is implemented in the form of an application developed using ArcObject and executed in ArcGis environment.

2. Materials and Methods

2.1. Calculation of safety factors

Hovland's method (Hovland 1977) is used to calculate the safety factor, it's an extension of the assumptions used in the two-dimensional ordinary method of slices also called OMS. The method is based on dividing the volume of moving soil into a specific number of parallelepiped columns. Inter-column forces acting on each column's base are presumed to be zero, however the normal forces and the shear forces are calculated as column weight components.

Hovland (1977) considered that the soil movement is oriented along the direction where the equilibrium of the system can be determined and that the 3D safety factor is defined as the ratio of the total available resistance along the slip surface, over the total stress mobilized along it. By dividing the soil mass above the failure surface into several vertical columns, the safety factor is determined, the X and Y lines are assumed perpendicular and located in the horizontal plane. ΔX and ΔY define the area of the soil column in the XY plane (Figure 1).
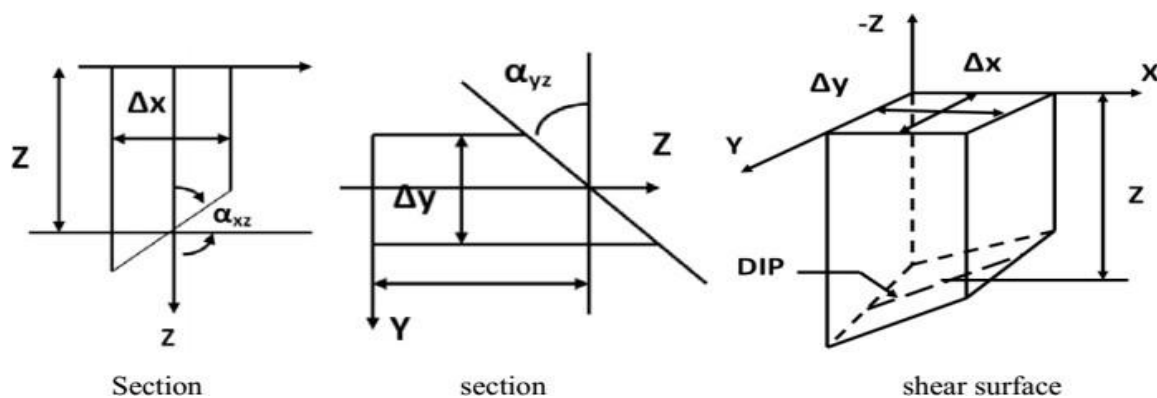

Figure 1 Three-Dimensional views of one soil column (Hovland, 1977).

Hovland's three-dimensional safety factor is given by:

$$Fs3D = \frac{\sum_x \sum_y \left[\dfrac{c'.sin\theta.\Delta x.\Delta y}{\cos\alpha_{xz}.\cos\alpha_{yz}} + (\gamma z \cos(Dip)\Delta x\Delta y - \alpha_w h_w \dfrac{sin\theta.\Delta x.\Delta y}{\cos\alpha_{xz}.\cos\alpha_{yz}})tan\varphi' \right]}{\sum_x \sum_y \gamma z \sin\alpha_{yz}\Delta x\Delta y} \tag{1}$$

In which α_{xz} and α_{yz} are Dip angles in their respective planes, and:

$$\cos(Dip) = \frac{1}{\sqrt{1 + \tan^2\alpha_{xz} + \tan^2\alpha_{yz}}}$$

W: Weight of the column, it is defined by $\gamma z\Delta X\Delta Y$.

A: Area of soil column, it is given by $A = \dfrac{\sqrt{1 - sin^2\alpha_{xz} - sin^2\alpha_{yz}}}{\cos\alpha_{xz}.\cos\alpha_{yz}}.\Delta x\Delta y$ with $sin\theta = \sqrt{1 - sin^2\alpha_{xz} - sin^2\alpha_{yz}}$

c' : Effective cohesion
φ' : Effective angle of shearing resistance
h_w: Water level depth to slide surface
γ : Unit weight of soil
z : Depth of failure surface below the surface
Dip : Aspect in GIS

2.2. Slope stability analysis using probabilistic approach

2.2.1. *Generation of random variables by Monte Carlo simulation*

In engineering, where the calculation model contains random variables and identifies groups of uncertain parameters that will substantially affect slope stability, probabilistic analysis is utilized (Gilbert et al 1998; Zhang et al 2010; Muthukumar et al 2022). Probabilistic analysis was performed by simulating random variables on the stability calculation. The initial safety factor was calculated using the Hovland method [7]. Several recalculations of the safety factor using Monte Carlo simulation were performed. For Monte Carlo simulation, two parameters (c' and φ') are chosen as random variables, and their random variables are assumed to be in uniform distribution. The random variable is calculated using the random variable Rnd, in the range of [0, 1], which is obtained by:

$$\beta_i = \alpha \beta_{i-1} Mod(m), \tag{2}$$
$$Rnd_i = \frac{\beta_i}{m}$$

Where α = a positive integer constant; m = the module; Rnd_i= the uniform distribution's random variable within the range [0,1]; each random variable Rnd_i can be obtained by providing an initial value of β_0. The random variable inside the range [a,b] is then computed as follows:

$$Xi = Rnd_i(b - a) + a \tag{3}$$

where X_i= the random variable within range of $[a, b]$.

3. Implementation of computation

Based on two approaches, a calculation model is implemented in the form of an application developed using ArcObject and executed in ArcGis environment. This model calculate the safety of a slope using a deterministic and probabilistic approach. The 3D minimum safety factor and a raster map of slope instability hazards constitute the results provided by the application in probabilistic approach. A flowchart of the application used in this study is shown in Figure 2.
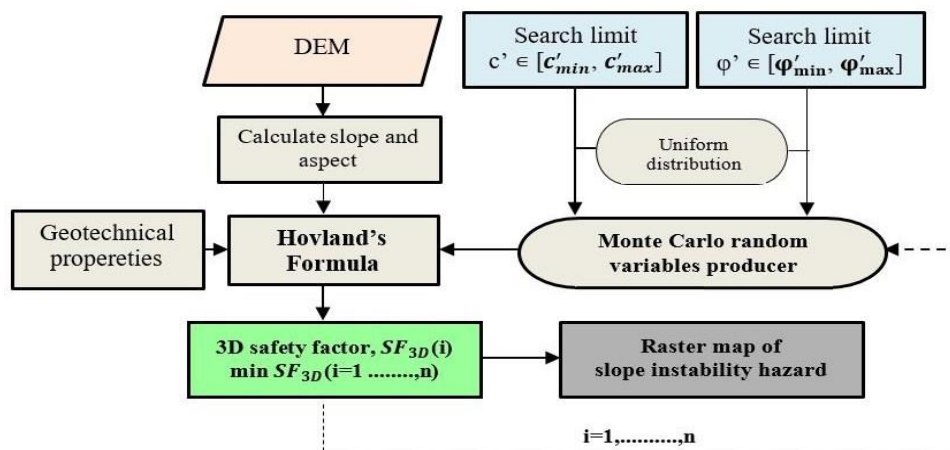

Figure 2 Flowchart of the application.

4. Result and discussion

4.1. *Calculation of slope stability taking into account of the variability of soil properties*

Two parameters were defined as random variable, the cohesion and the friction angle of the soil. The safety factor calculation is based on the Monte-Carlo simulation, which consists in random sampling of the input parameters, and then a deterministic calculation is carried out for each realization. For this, the minimum and maximum values of sampling interval as well as the maximum number of iterations must be defined. At each iteration, the geotechnical properties are generated then the calculation of the safety factor is performed. The calculation stops when the maximum number of iterations is reached. An example of a stability calculation considering the variability of soil parameters is shown in Figure 3.

4.2. *Identification of slope instability: case study*

Using Monte-Carlo simulation, the developed application generates half-spheres of radius R that vary randomly according to a uniform distribution. At each iteration, the safety factor is calculated. The calculation stops after the maximum number of iterations defined by the user is reached. The 3D safety factors for each iteration as well as a Raster map representing the slope instability constitute the results provided by the application.

In order to identify the slope instability hazard, the proposed method was applied to the slope located about 34 Km East of the Tangier city (northern Morocco) on the highway connecting Tangier to Tangier Med Port (Figure 4). The mechanical properties of the studied slope are shown in Table 1.
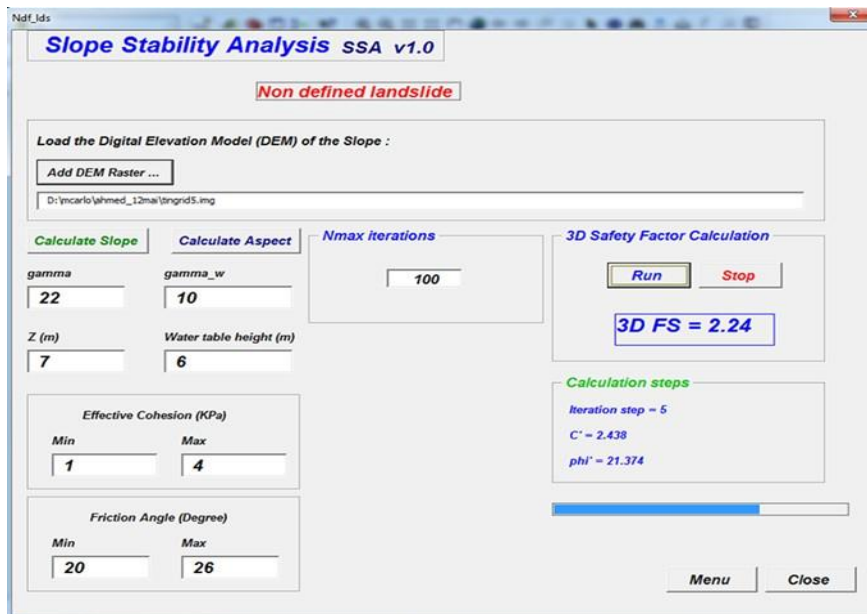

Figure 3 Stability calculation considering the variability of soil parameters.
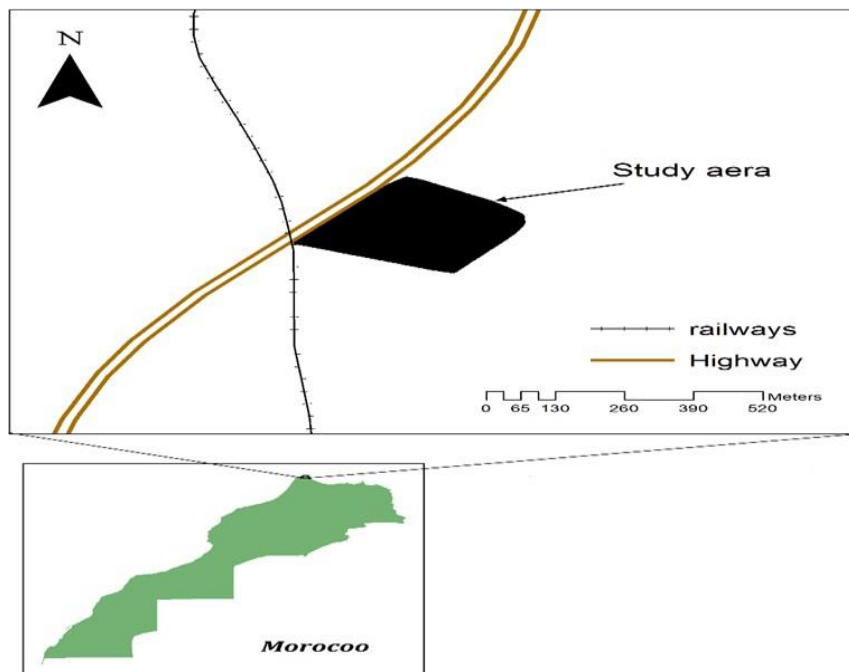

Figure 4 Localization map of the study area.

A Raster map representing the slope instability of the studied slope is shown in Figure 5. The analysis results show that the safety factor varies between 0.8 and 2.44; for the stability of the slope to be considered safe, the safety factor must be greater than 1. We note that the red color indicates unstable areas; it is clear that the unstable areas coincide with zones with low mechanical characteristics.

Table 1 Mechanical properties of the studied slope.

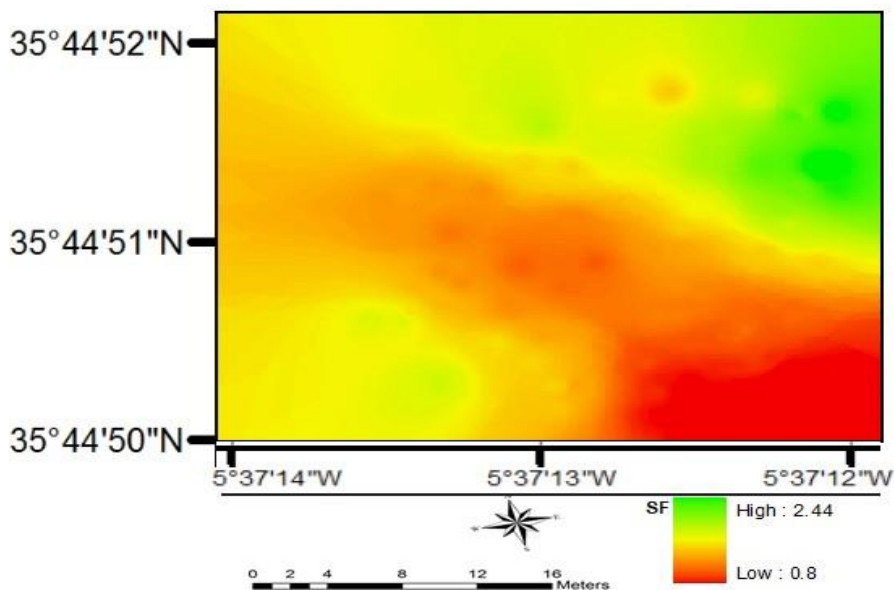| Soil	Effective cohesion (kPa)	Friction angle (°)	Unit weight (KN$/m^3$)
Weathered pelite	3	24	22
Pelite (bedrock)	6	26	20

Figure 5 Safety factor map of part of the study area obtained by the application.

4.3. Identification of slope instability: case study

An example of homogeneous slope was chosen from the literature (Greco 1996; Malkawi et al 2001; Xie et al 2004) to check and evaluate the applicability of the suggested method (Figure 6). The studied slope area has dimensions of 25*40 meters, and the gradient of the slope is approximately 26 degrees. The geotechnical properties are presented in Table 2.

Table 2 Geotechnical properties of homogenous slope.

Effective cohesion (kPa)	Friction angle (°)	Unit weight of soil (KN$/m^3$)
9.8	10.7	17.64

Different researchers have solved the case of a homogeneous slope. Greco (1996) used pattern search methods and Monte Carlo to investigate the critical slip surface. Malkawi et al (2001) used the Monte Carlo random walk approach to determine the critical slip surface. In addition, to find the 3D critical slip surface, Xie et al (2004) employed a Monte Carlo random simulation method, considering the initial slide as the lower portion of an ellipse.
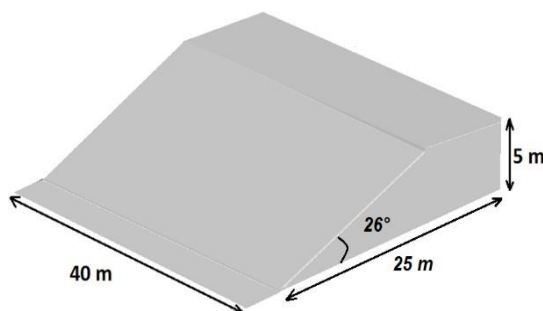

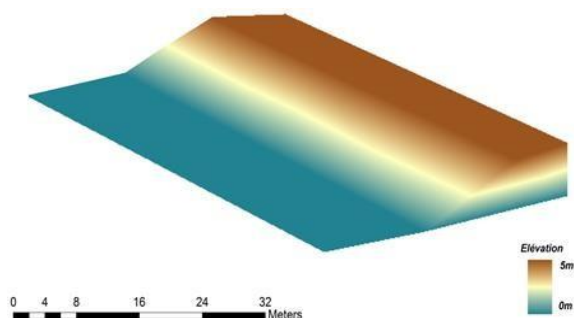**Figure 6** Homogenous slope geometry.

Figure 7 DEM of homogenous slope.

Using the same geotechnical parameters and profile as in the 2D researches, the homogeneous slope is generalized to a straightforward 3D problem (Figure 7).The results obtained in comparison with those 2D studies by different authors with the 3D result are shown in Table 3.

<p align="center">Table 3 Minimum safety factors of homogeneous slope.</p>

Method	Safety factor
1) Greco (1996)	
Monte Carlo	1.327 (2D)
Pattern search	1.327 (2D)
2) Malkawi et al (2001)	
Monte Carlo	1.238 (2D)
3) Xie et al (2004)	
Monte Carlo	1.42 (3D)
4) This study (3D)	
Monte Carlo	1.36 (3D)

We note that the 3D safety obtained by minimization procedures is higher than those of 2D results. The origin of this difference is probably comes from the assumptions and limitations made on the 2D and 3D methods of stability calculation although the authors do not explain this point.

5. Conclusions

One of the main goals of this paper is probabilistic analysis. The combination of Hovland's method and random soil strength parameters by Monte Carlo simulation enabled us to calculate the safety factor of a slope based on the distribution of random soil strength parameters.

An example of a homogeneous slope demonstrates the efficacy of the proposed method. In order to calculate the 3D safety factor and compare it to the 2D results, the homogeneous slope is extended to 3D. Results show that the 3D safety factor obtained by proposed method is about 11% greater than those of the 2D results; this increase in safety factor is due to the proper inclusion of end effects and changing directional strength, which cannot be included in a 2D analysis.

This study provides a methodology for analyzing homogeneous slopes using a number of independent random variables. In further studies, more complex cases can be carried out, such as adding other random variables and considering the influence of groundwater table and heterogeneous soil condition for the analysis.

Ethical considerations

Not applicable.

Conflict of Interest

The authors declare that they have no conflict of interest.

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